Introduction to Nastran SOL 200 Design Sensitivity and Optimization

PRESENTED BY: CHRISTIAN APARICIO

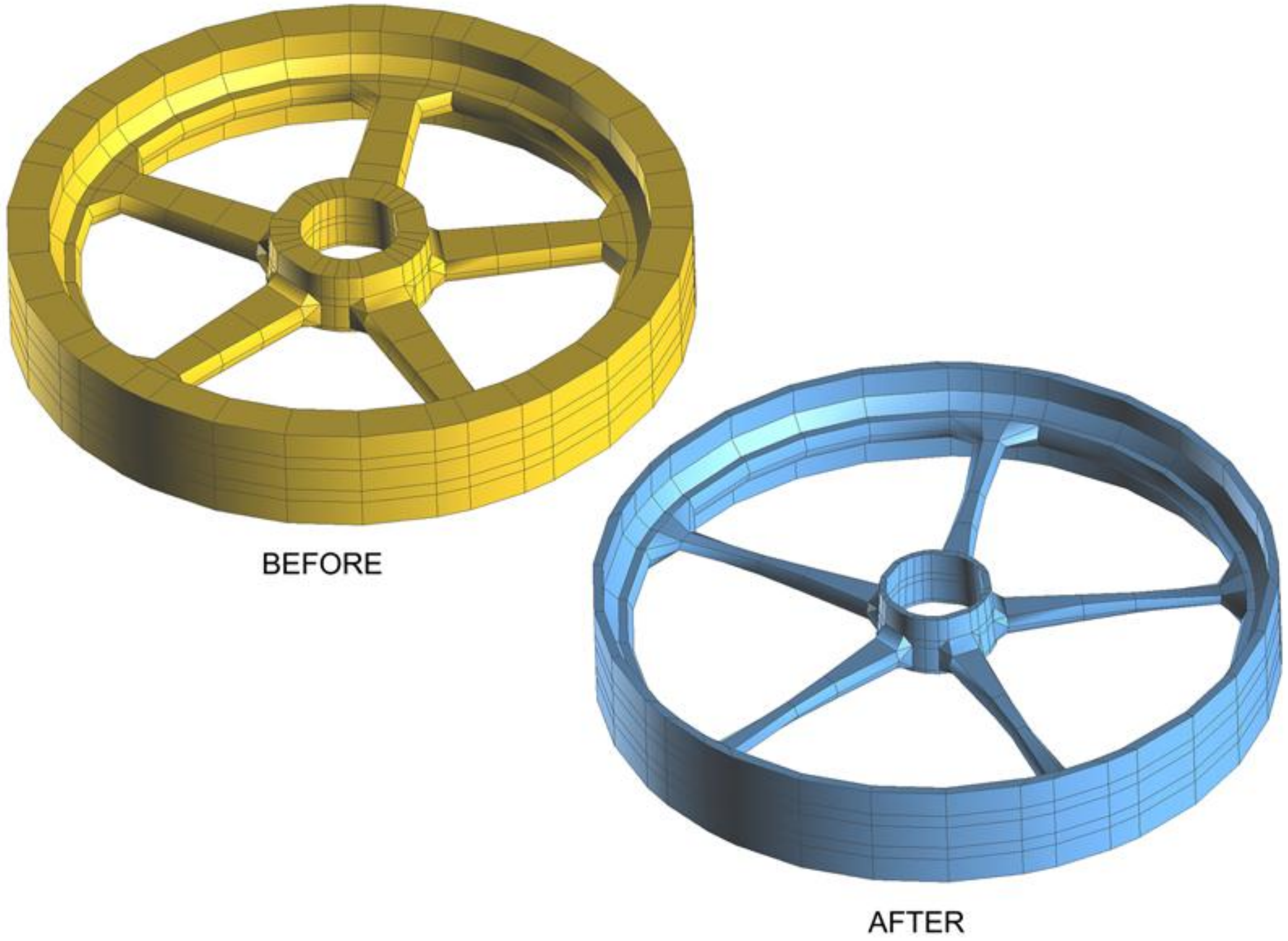
Motivation

It is my intent that after you read this guide, you will be one step closer towards performing a unique procedure only a limited number of engineers can do.

That is, optimizing structures automatically with Nastran SOL 200.

Kind Regards,

Christian Aparicio



Shape Optimization is used to find optimum boundaries of this pulley

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What is optimization?

Suppose we define and plot the function below:

$$f(x_1, x_2) = x_1^2 + x_2^2$$

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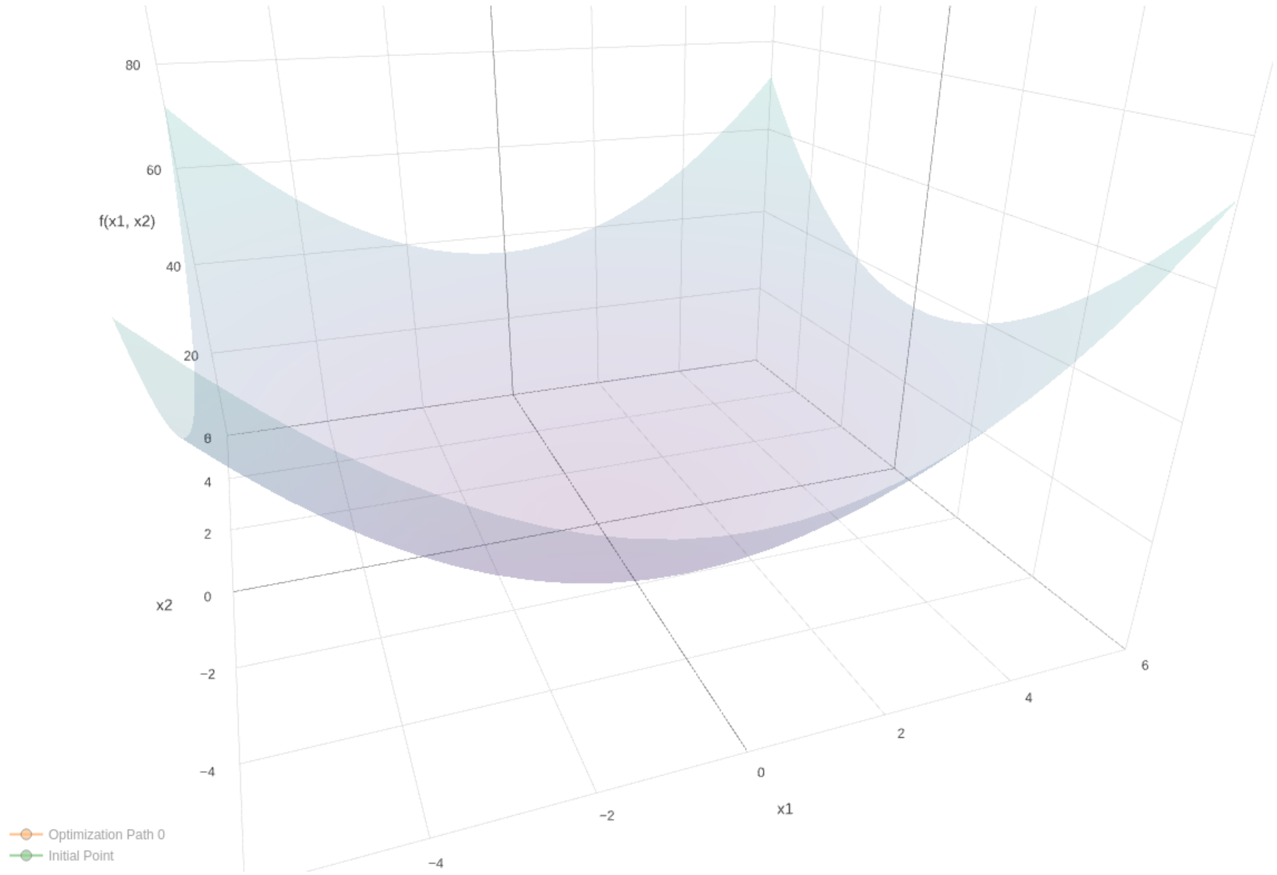
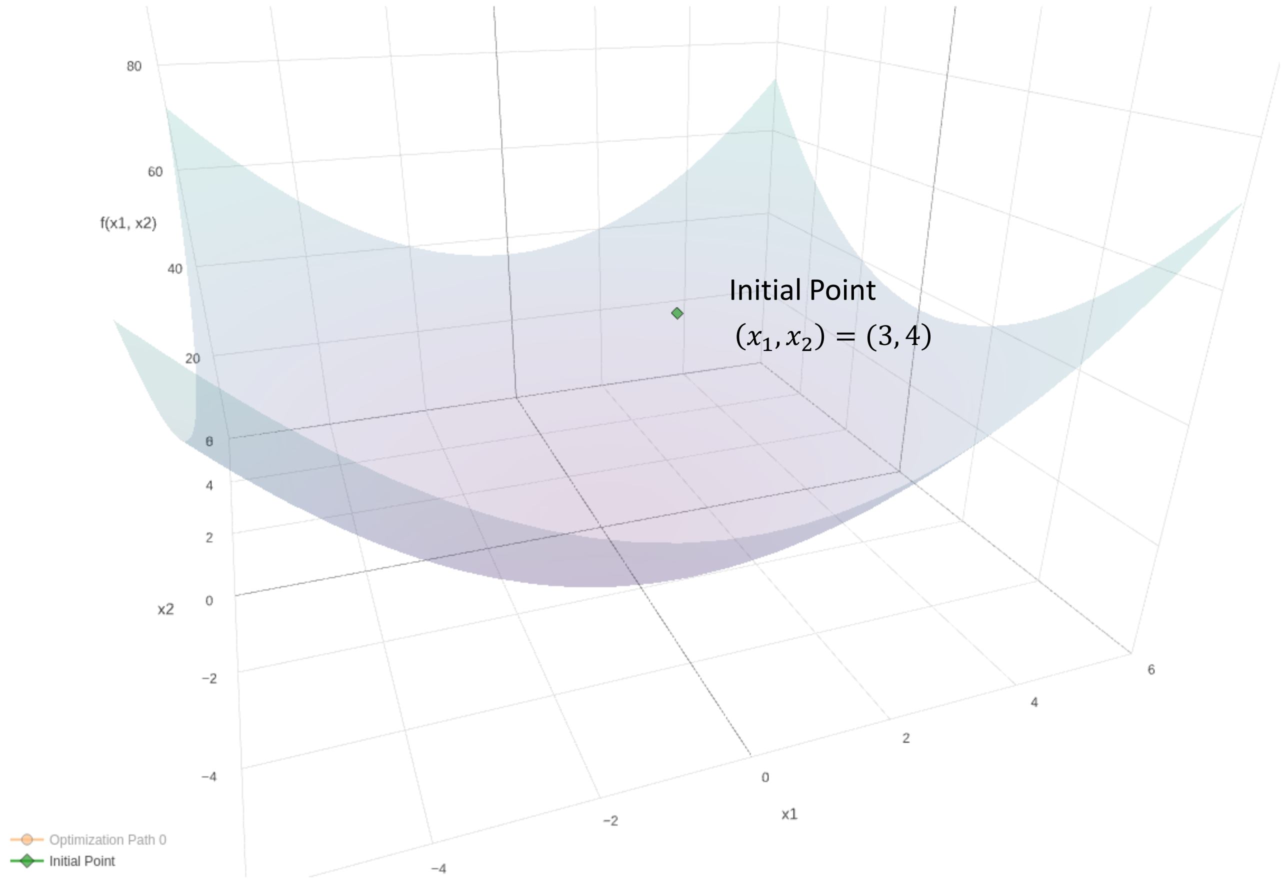


We start at point: $(x_1, x_2) = (3, 4)$

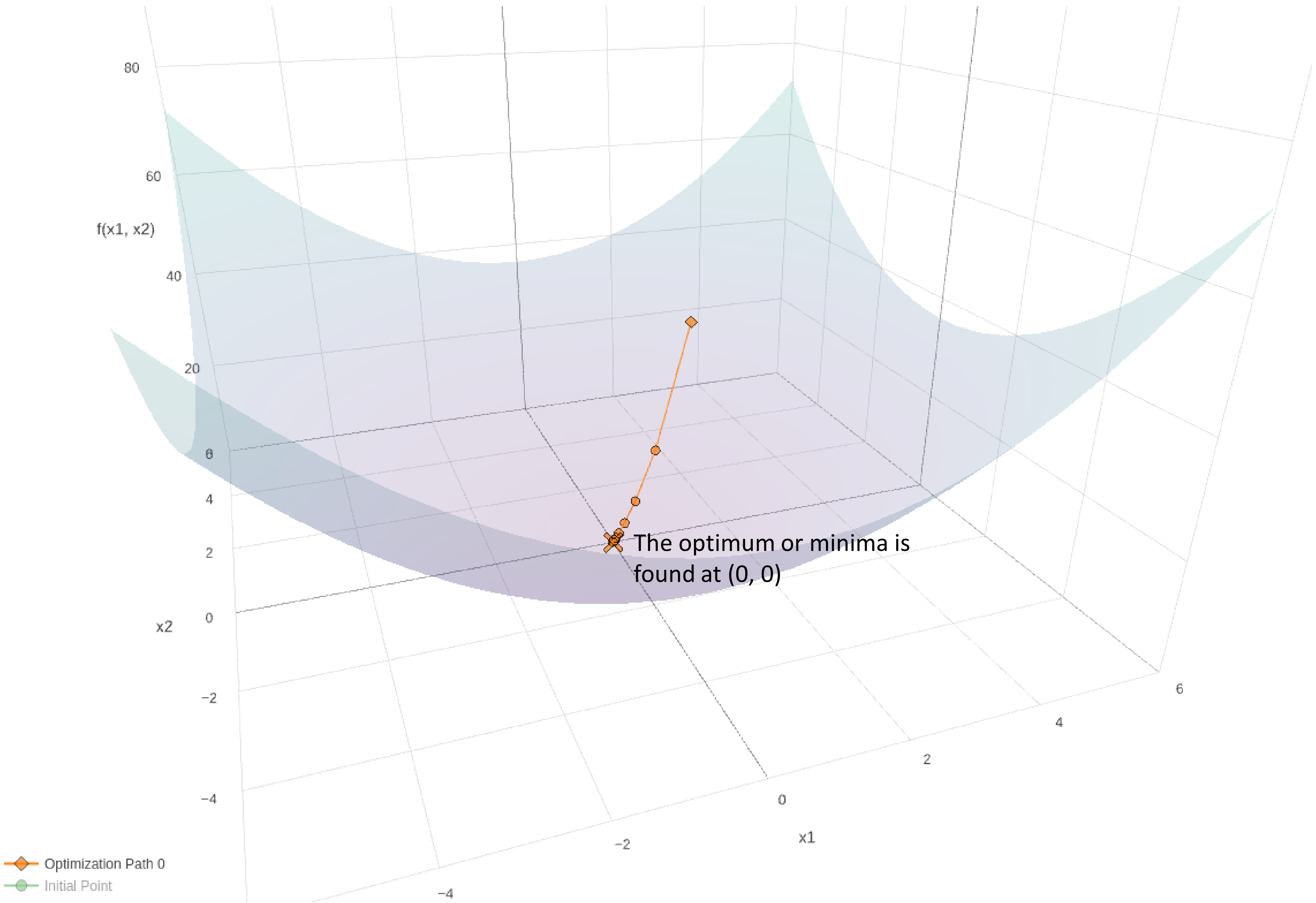
The objective is to find the minima of $f(x_1, x_2)$

$$f(x_1, x_2) = x_1^2 + x_2^2$$



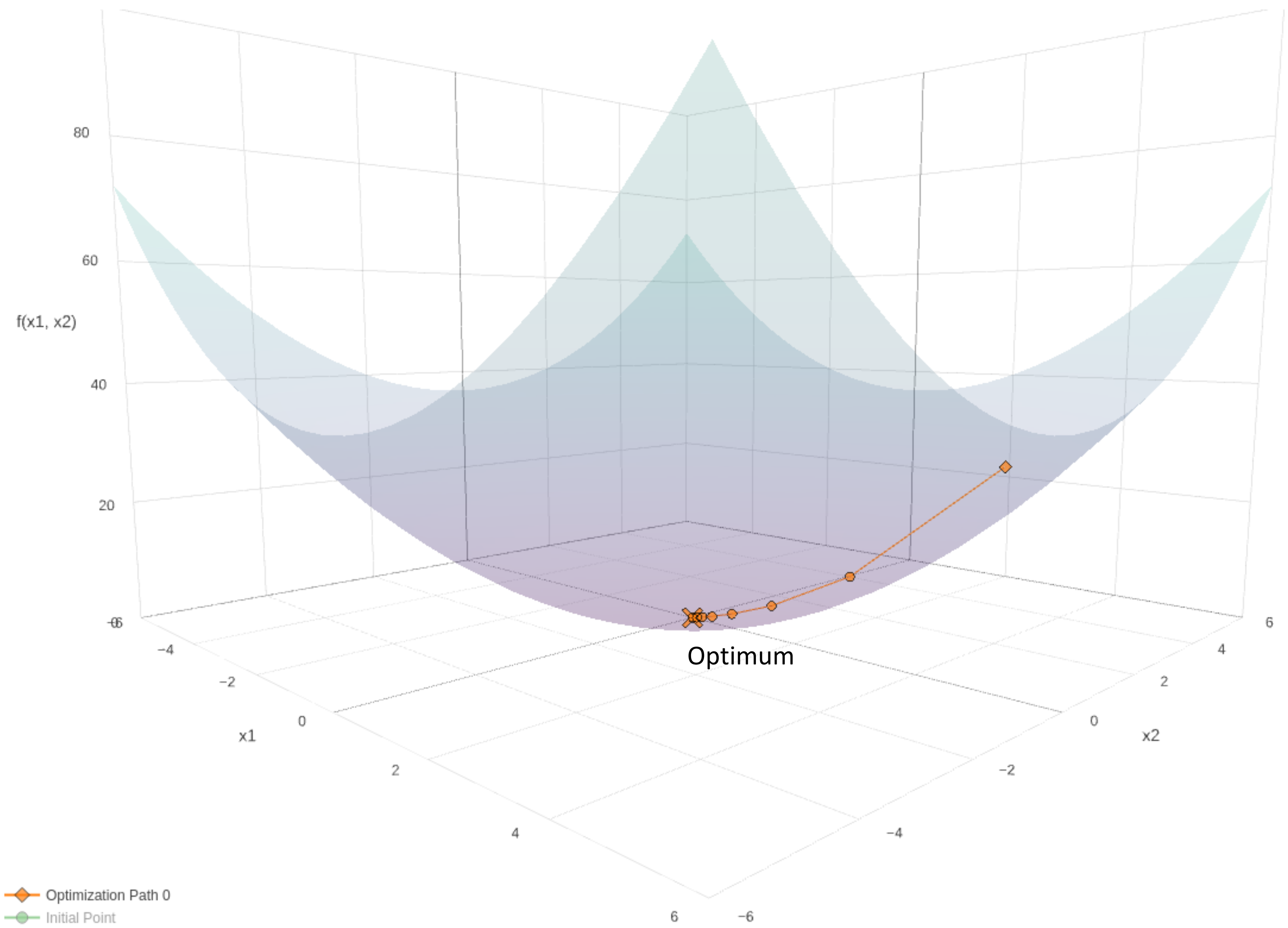
The optimizer in Nastran SOL 200 finds the minima at (0, 0).

$f(x_1, x_2) = x_1^2 + x_2^2$



Optimization is the process of finding minima or maxima of functions.

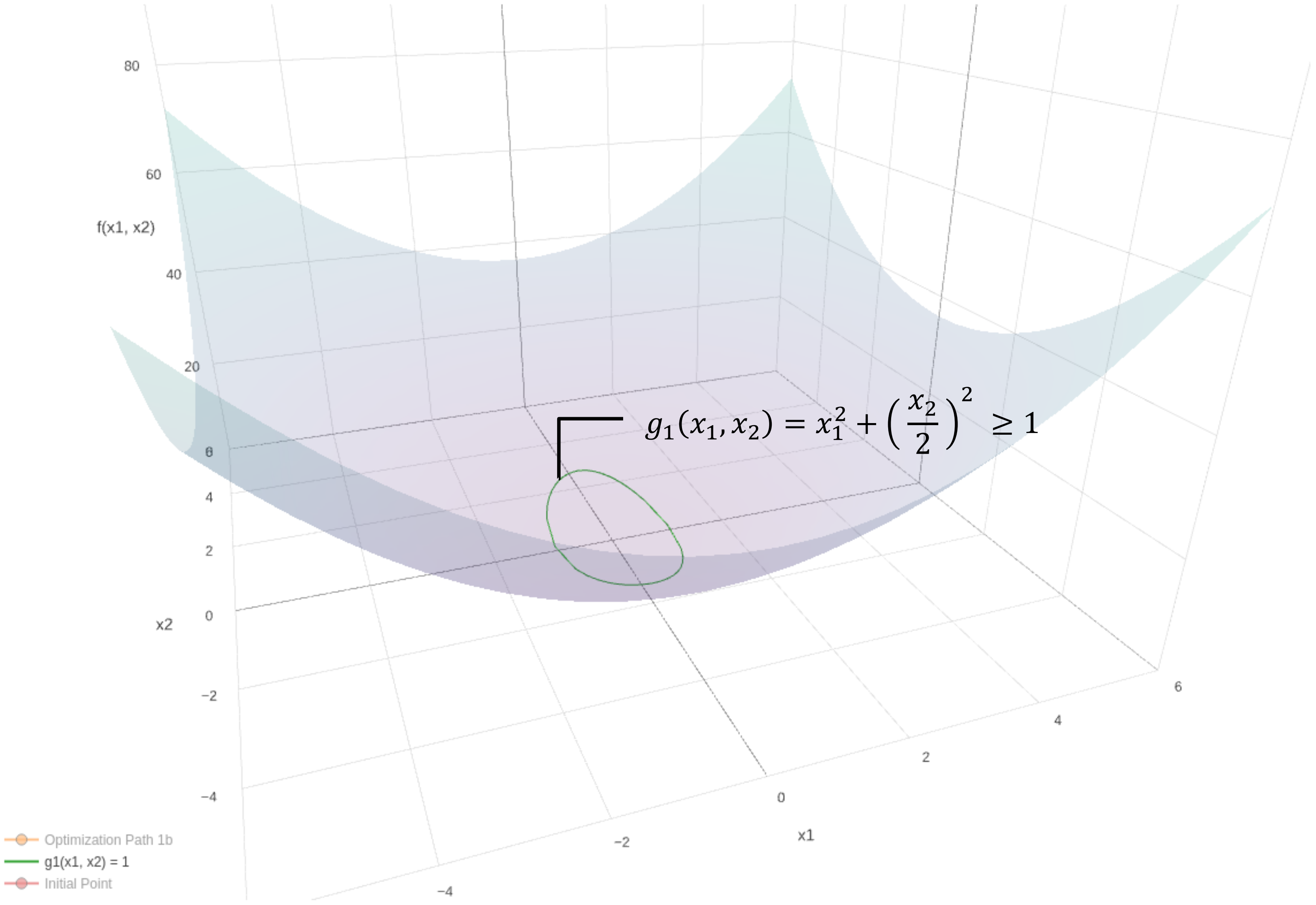
$$f(x_1, x_2) = x_1^2 + x_2^2$$



Let’s continue the previous example and add this constraint:

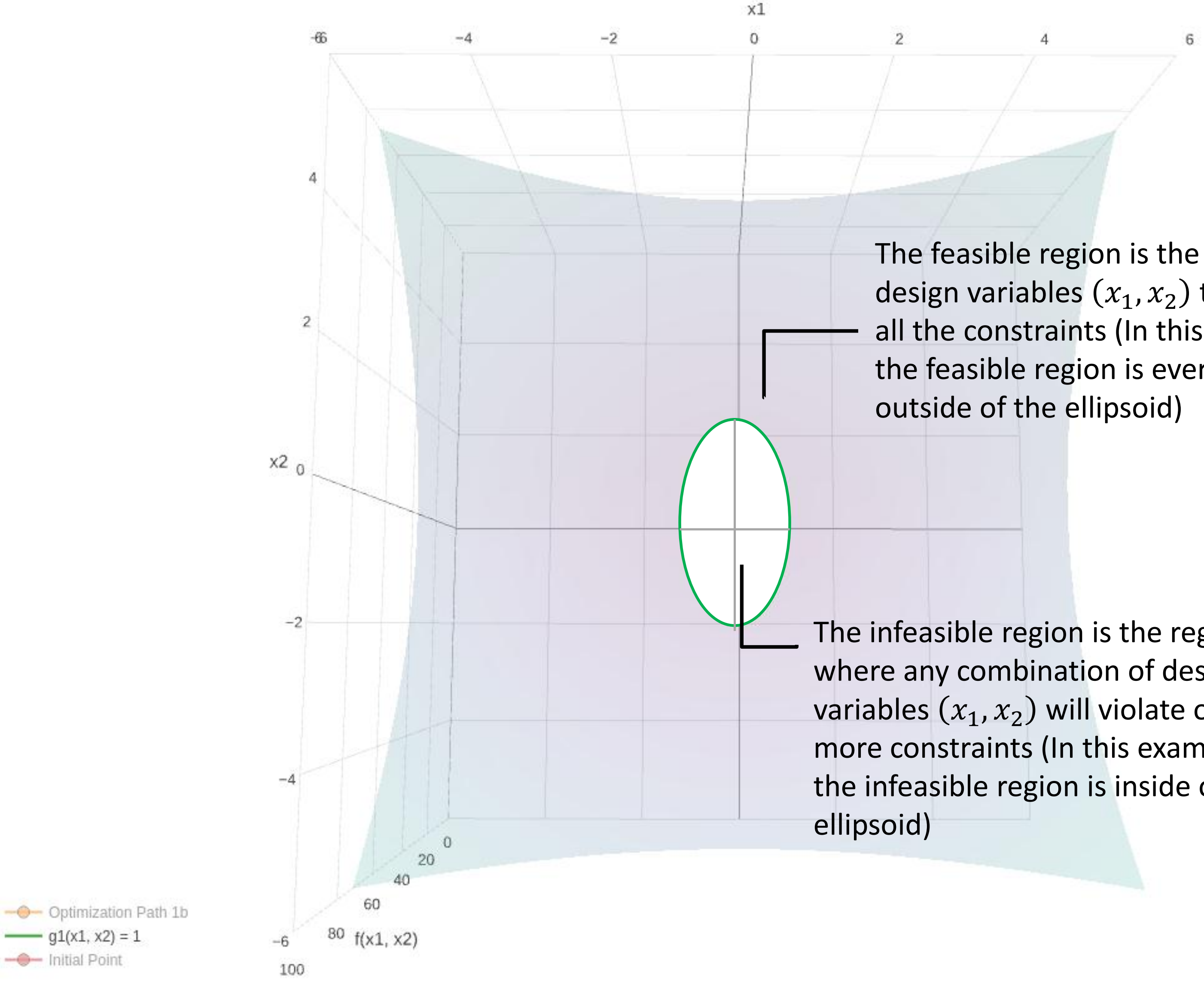
$$g_1(x_1, x_2) = x_1^2 + \left(\frac{x_2}{2}\right)^2 \geq 1$$

$$f(x_1, x_2) = x_1^2 + x_2^2$$



What does the optimizer do in this scenario? See next slide.

$$f(x_1, x_2) = x_1^2 + x_2^2$$

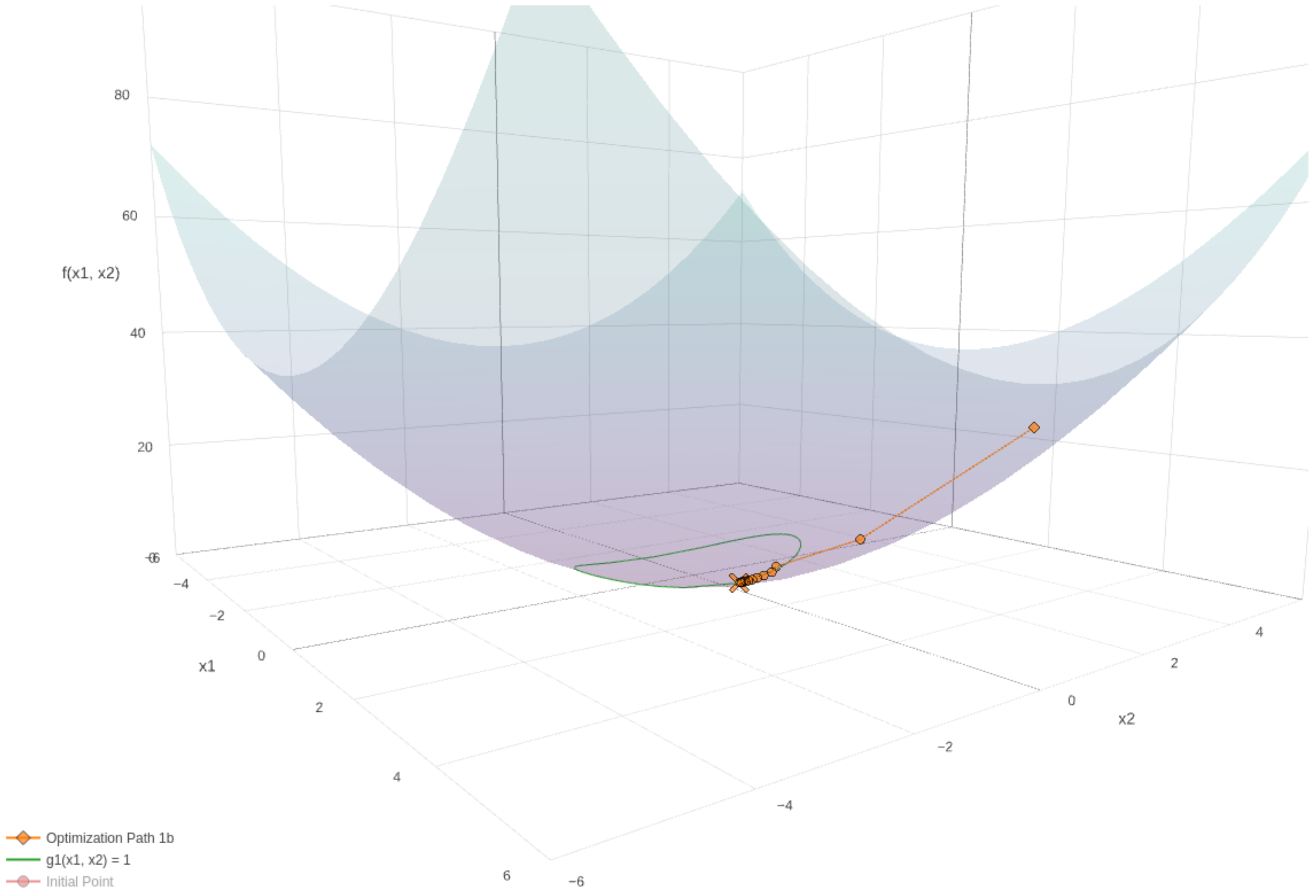


The feasible region is the set of design variables (x_1, x_2) that satisfy all the constraints (In this example the feasible region is everything outside of the ellipsoid)

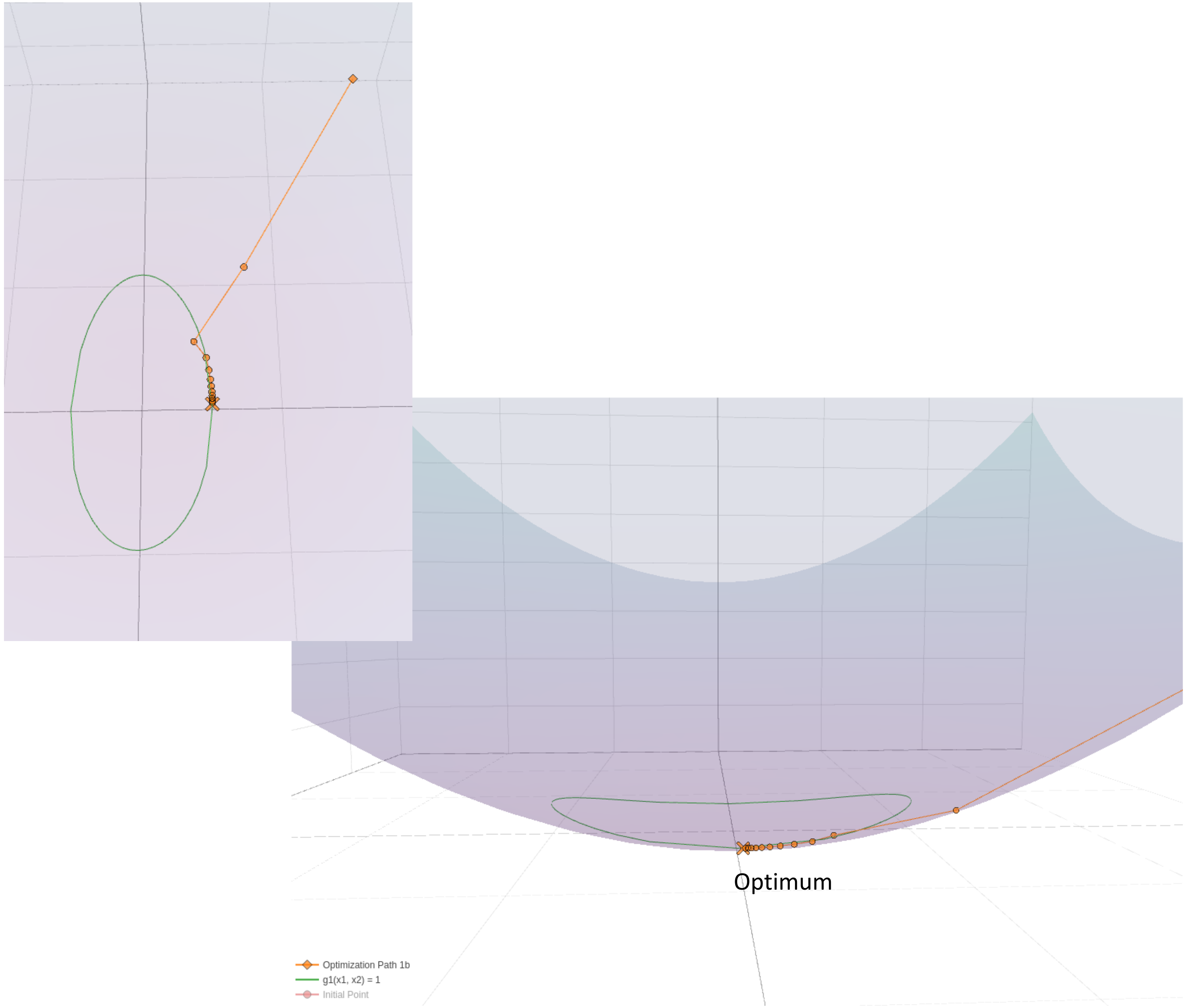
The infeasible region is the region where any combination of design variables (x_1, x_2) will violate one or more constraints (In this example, the infeasible region is inside of the ellipsoid)

The optimizer will move towards the global minima

$$f(x_1, x_2) = x_1^2 + x_2^2$$



Once it reaches the constraint, the optimizer will move along the constraint until the minima is found



What is the optimization problem statement?

The optimization problem statement is composed of 3 items:

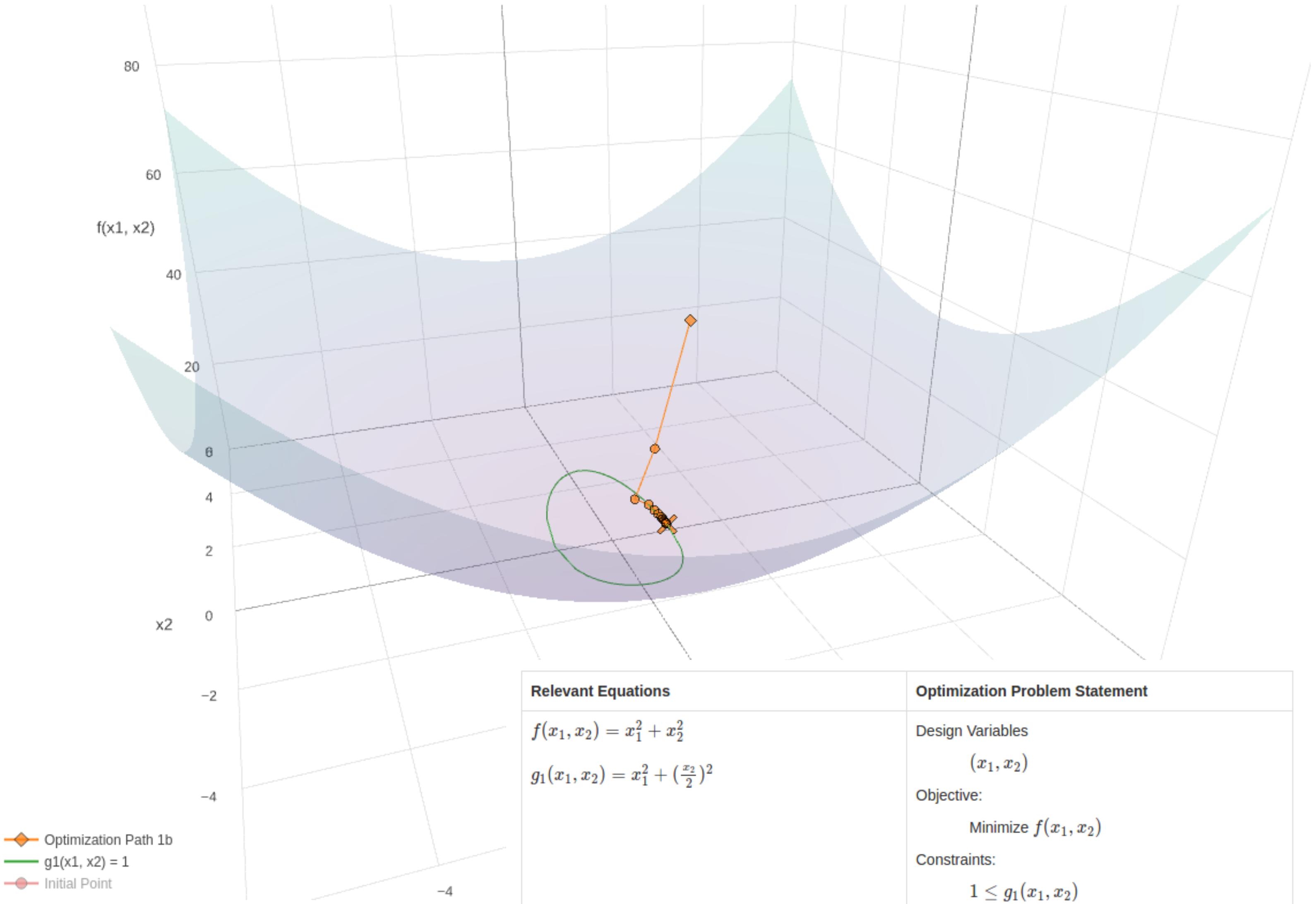
1. Design Variables – What parameters are allowed to vary?
2. Design Objective – What quantity is to be minimized or maximized?
3. Design Constraints – What quantities are constrained?

This is the optimization problem statement for the last example

Relevant Equations	Optimization Problem Statement
$f(x_1, x_2) = x_1^2 + x_2^2$ $g_1(x_1, x_2) = x_1^2 + (\frac{x_2}{2})^2$	Design Variables (x_1, x_2) Objective: Minimize $f(x_1, x_2)$ Constraints: $1 \leq g_1(x_1, x_2)$

It is important to always have the optimization problem statement drafted before performing an optimization

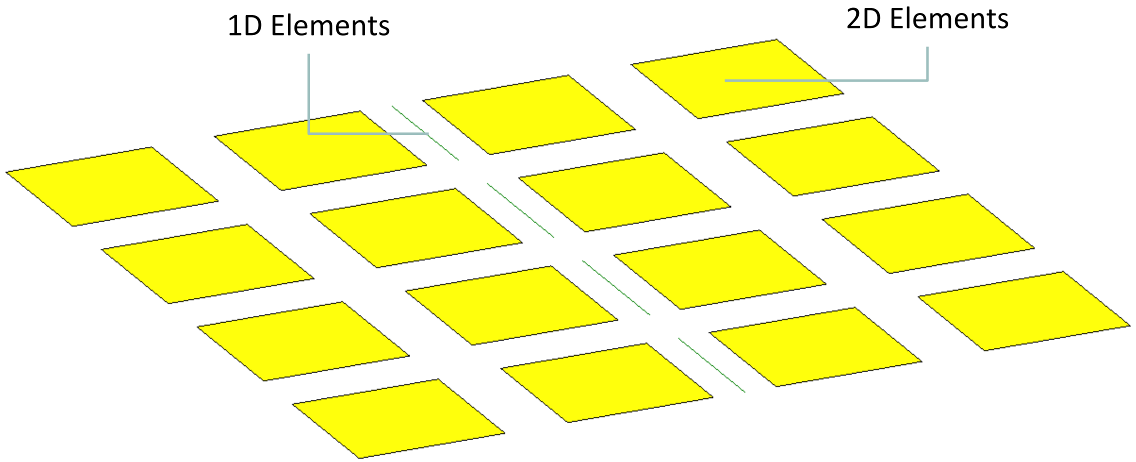
$$f(x_1, x_2) = x_1^2 + x_2^2$$



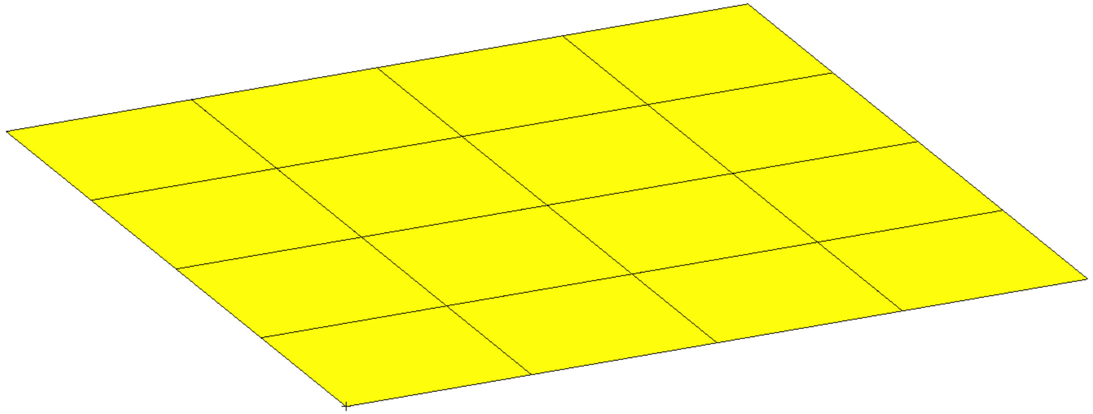
Relevant Equations	Optimization Problem Statement
$f(x_1, x_2) = x_1^2 + x_2^2$ $g_1(x_1, x_2) = x_1^2 + (\frac{x_2}{2})^2$	Design Variables (x_1, x_2) Objective: Minimize $f(x_1, x_2)$ Constraints: $1 \leq g_1(x_1, x_2)$

What is size optimization?

Consider the example finite element model to the right.

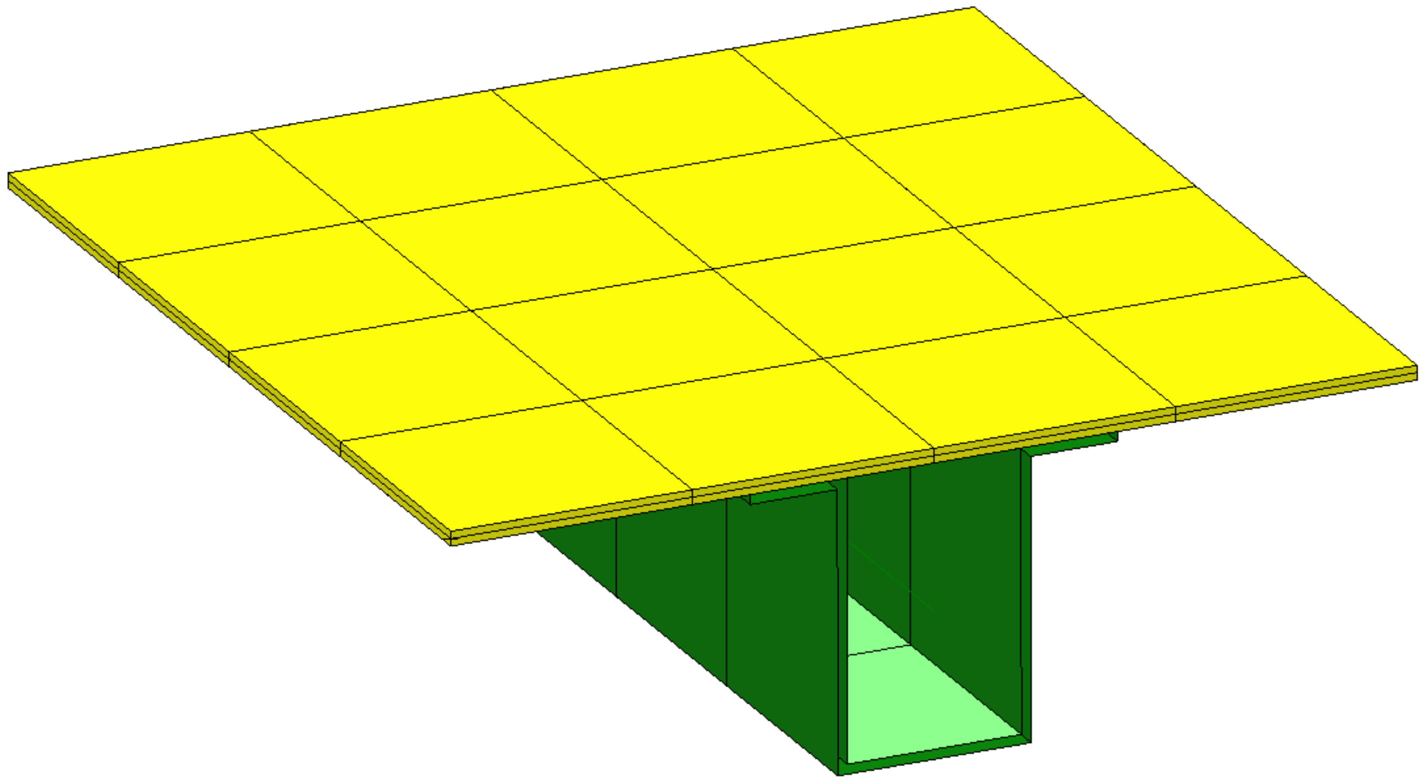


Disassembled View



Assembled View

After assigning parameters such as thickness to the 2D elements and beam cross section dimensions to the 1D elements, the structures looks as shown on the right.

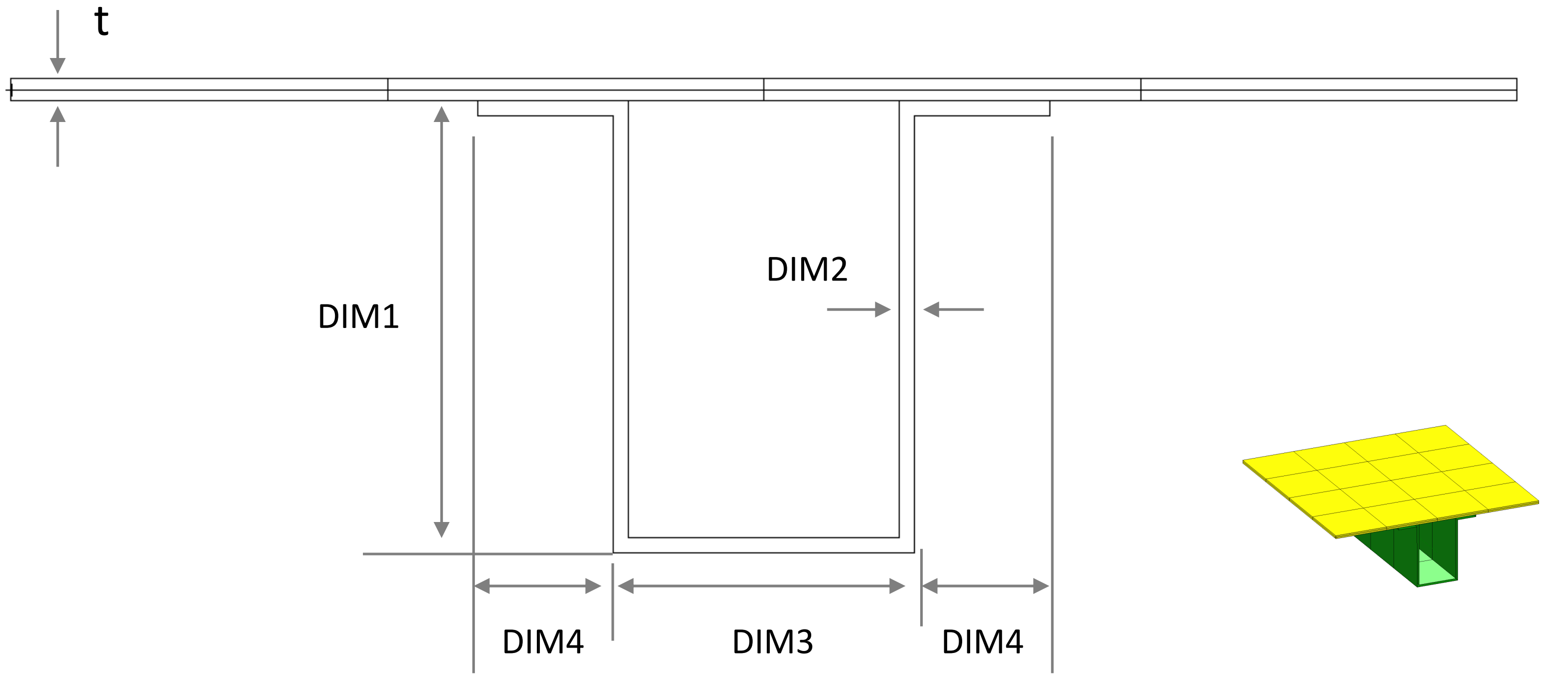


Size optimization is the process of setting parameters as design variables.

For example, the thickness and dimension 1 and 2 can be set as design variables.

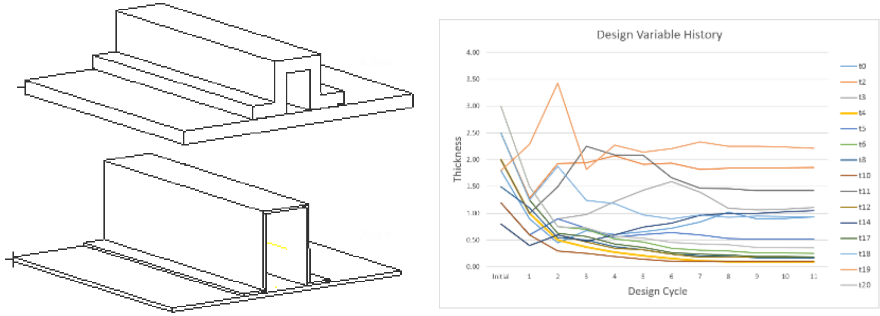
- x_1: t, thickness
- x_2: DIM1
- x_3: DIM2

Other parameters that can be set as design variables include: Young's modulus or density of a material. The complete list of parameters that can be set as design variables is extensive.

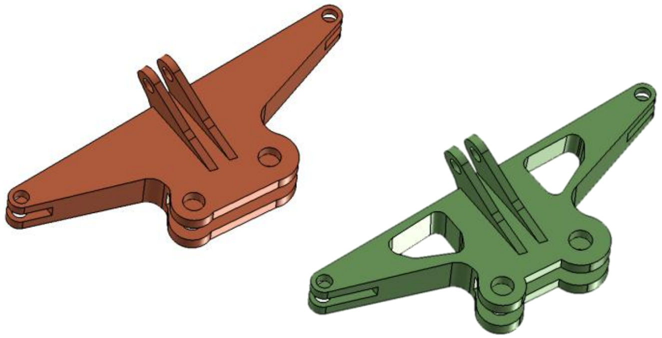


Note that many optimization types exist. To the right are some of the many types.

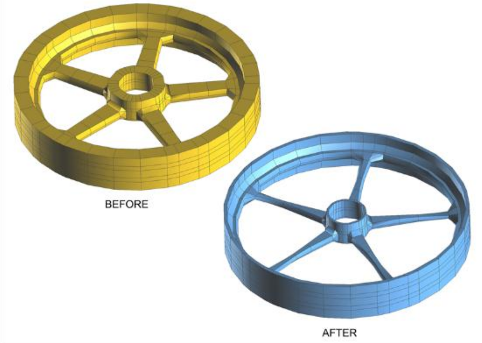
The remainder of this guide will only apply to size optimization.



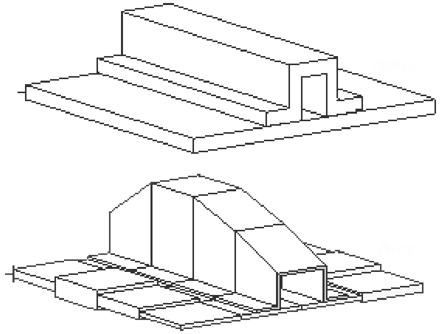
Size Optimization



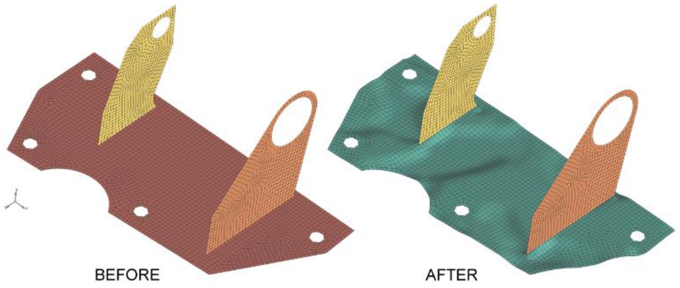
Topology Optimization



Shape Optimization



Topometry Optimization



Topography Optimization

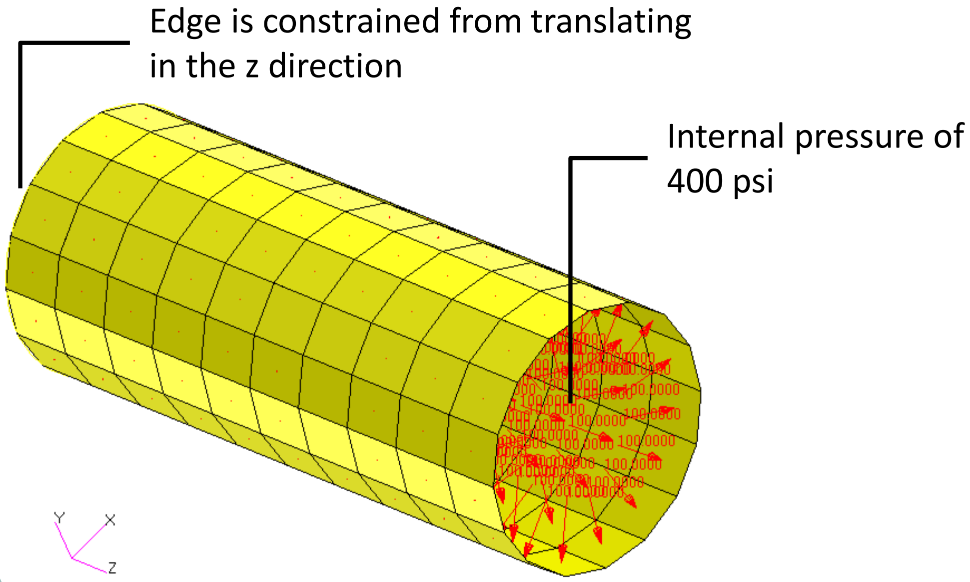
Optimization Examples with Nastran SOL 200

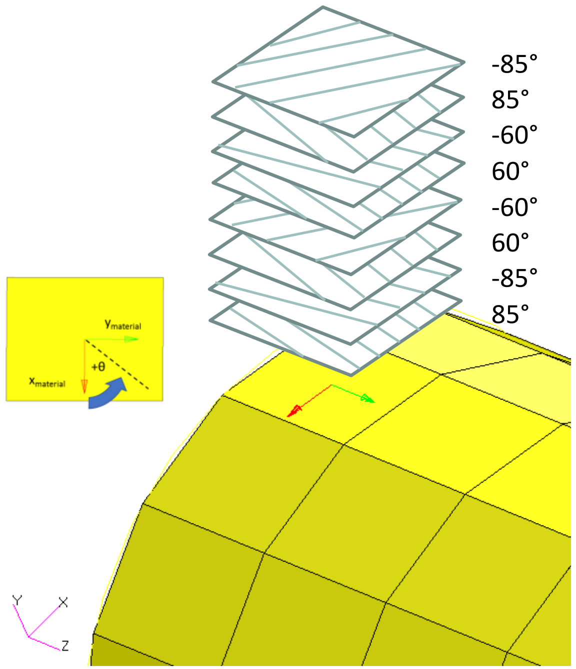
Example 1 - Optimization of a composite laminate

A cylindrical tube is composed of a composite laminate with the following layup: [85/-85/60/-60/60/-60/85/-85]. Each ply has a thickness of .01 inches.

Loading and Constraints

The tube is allowed to freely expand radially outward. An internal pressure of 400 psi is applied.





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Example 1 - Optimization of a composite laminate

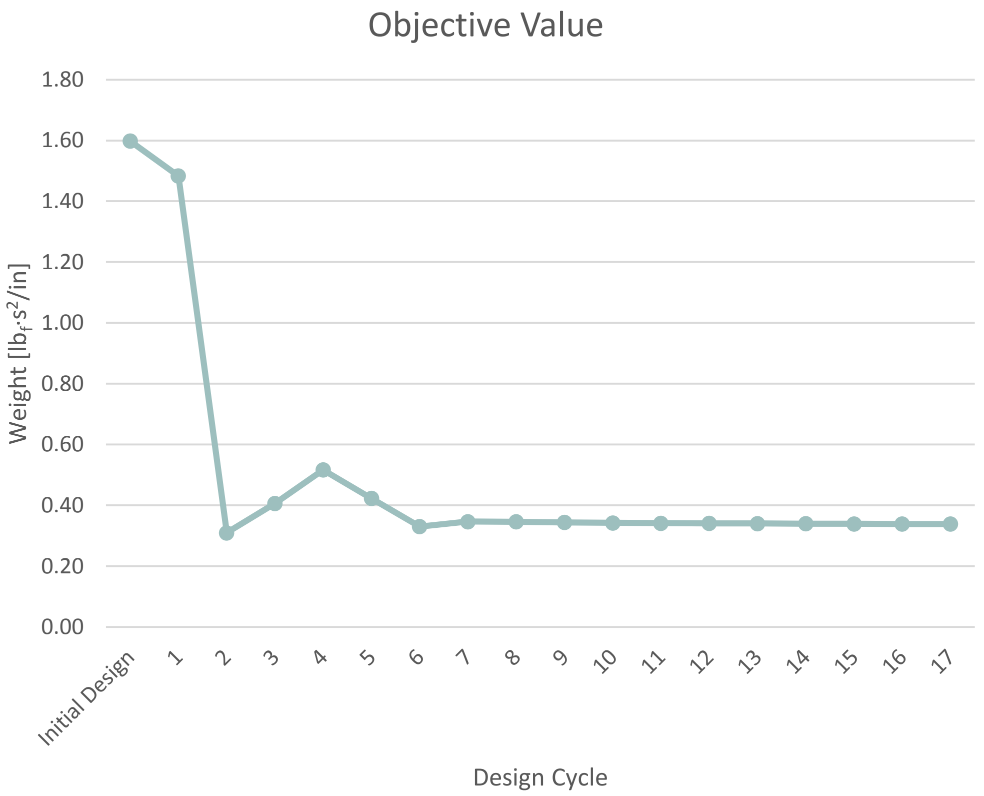
Optimization Problem Statement

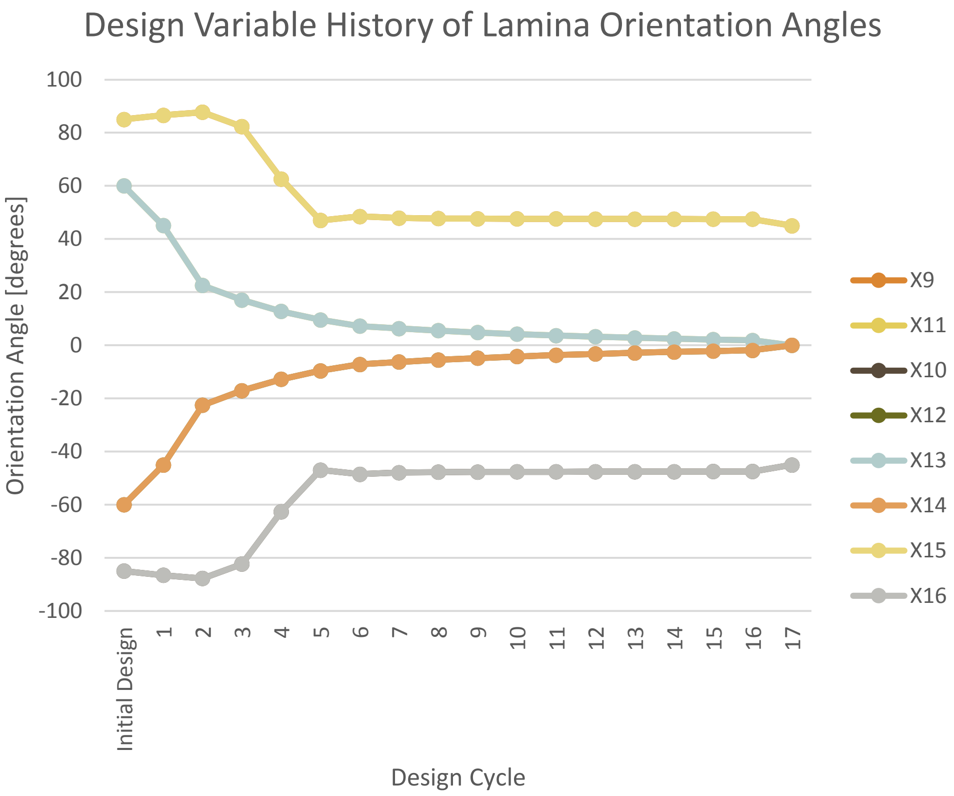
- Design Variables:
  - Thickness Variables
    - x1, x2, x3, x4, x5, x6, x7, x8 correspond to the thickness of lamina 1 through 8, respectively
    - Each thickness variable shall be equal to x1
    - x1 is allowed to range between .001 and 10.
  - Orientation Variables
    - x9, x10, x15, x16 correspond to the outer layer pair angles, i.e. 85 degree plies
    - x11, x12, x13, x14 correspond to the core pair angles, i.e. 45 degree plies
    - The orientation angles are allowed to vary between -90. and 90 degrees
    - Laminas 1, 2, 7, 8 should have the same orientation angle, but with opposing signs
    - Laminas 3, 4, 5, 6 should have the same orientation angle, but with opposing signs
    - The orientation angles are only allowed to be in 5 degree increments, e.g. 90, 85, 80… -80, -85, -90
- Design Objective
  - Minimize the weight
- Design Constraints
  - The strength ratio for each lamina shall be less than .9

Optimization Results

- Initial Design
  - Mass: 1.60 $lb_f \cdot s^2/in$
  - Layup: [85/-85/60/-60/60/-60/85/-85]
  - Ply Thicknesses: .0100 in.
  - Max strength ratio: 21.0
- Optimized Design
  - Mass: .3386 $lb_f \cdot s^2/in$
  - Layup: [45/-45/0/0/0/0/45/-45]
  - Ply Thicknesses: .0021 in.
  - Max strength ratio: .9

Example 1 - Optimization of a composite laminate





Example 2 - Model Matching / System Identification / Correlation to Experiment

A beam, fixed on one end, has a circular cross section with a radius of 2 in. The length of the beam is 30 in. Experiment has revealed the expected mode shapes for modes 1 and 3. A modes analysis of the finite element (FE) models shows a discrepancy between the FE model and experiment.

Optimization is used to find a radius of the cross section that will produce FE results comparable to experiment.

Experimental Results

	Mode 1	Disp.	Mode 3	Disp.
Node	Component	Value	Component	Value
3	z or 3 direction	0.0143	x or 1 direction	0.1204
6	z or 3 direction	0.1741	x or 1 direction	0.5431
9	z or 3 direction	0.6381	x or 1 direction	0.9216

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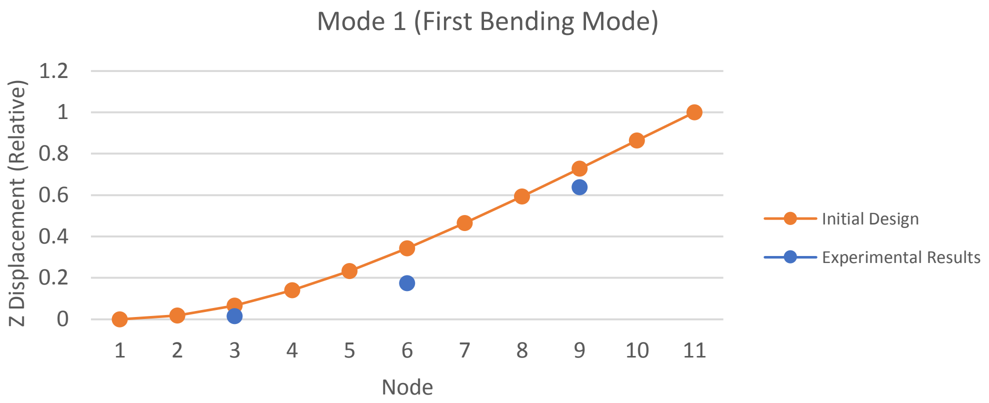
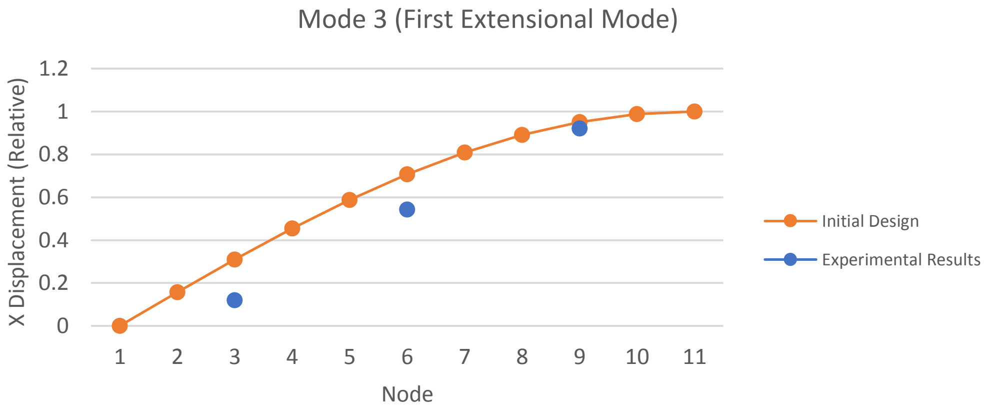
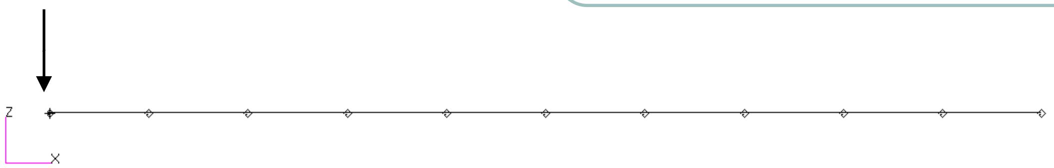




Fixed



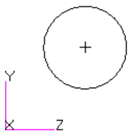
Length: 30 in

$E = 1 * 10^7$ psi

v = .3

Density = .01 $lb_f * s^2 / in^4$

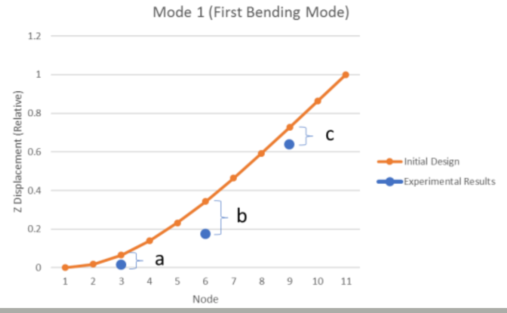
Radius: 2in



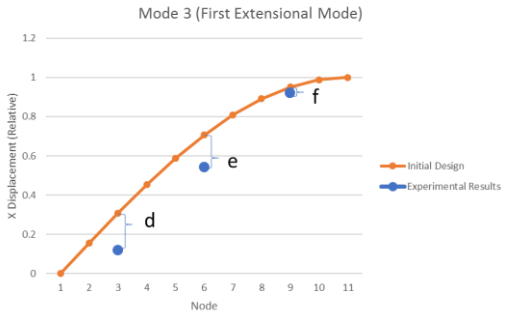
Example 2 - Model Matching / System Identification / Correlation to Experiment

Optimization Problem Statement

- Design Variables:
  - x1: The radius of 3 elements is allowed to vary between .1 and 10. inches.
- Design Objective
  - Experimental data is available regarding the 1st mode shape
  - The objective is to minimize the root sum of squares (RSS) between the experiment and FE results
    - f = RSS = sqrt ($a^2 + b^2 + c^2$)
      - a = (.0143 – r801)
      - b = (.1741 – r802)
      - c = (.6381 – r803)
      - r801, r802, r803 are the z displacements at nodes 3, 6, 9, respectively, for mode 1

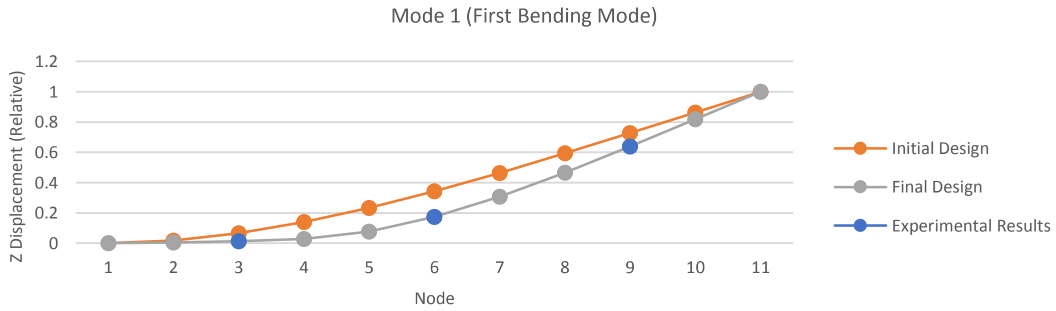


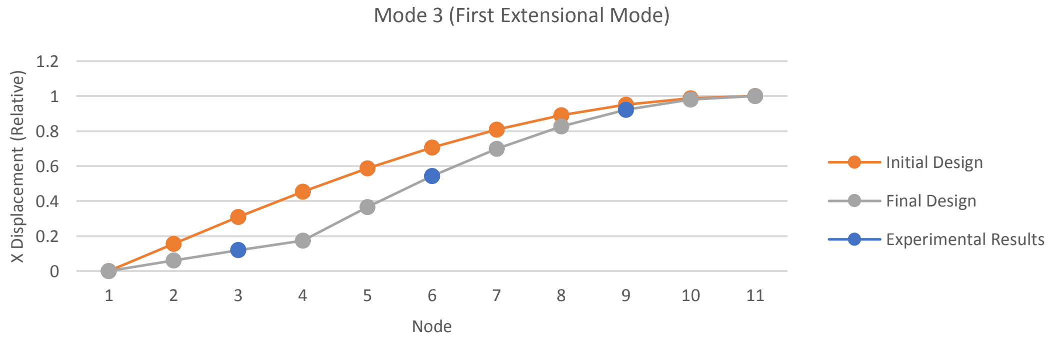
- Design Constraint
  - The RSS of mode 3 is to be less than .002
    - g1 = RSS = sqrt ($d^2 + e^2 + f^2$)
      - d = (.1204 – r501)
      - e = (.5431 – r502)
      - f = (.9216 – r503)
      - r501, r502, r503 are the x displacements at nodes 3, 6, 9, respectively, for mode 3



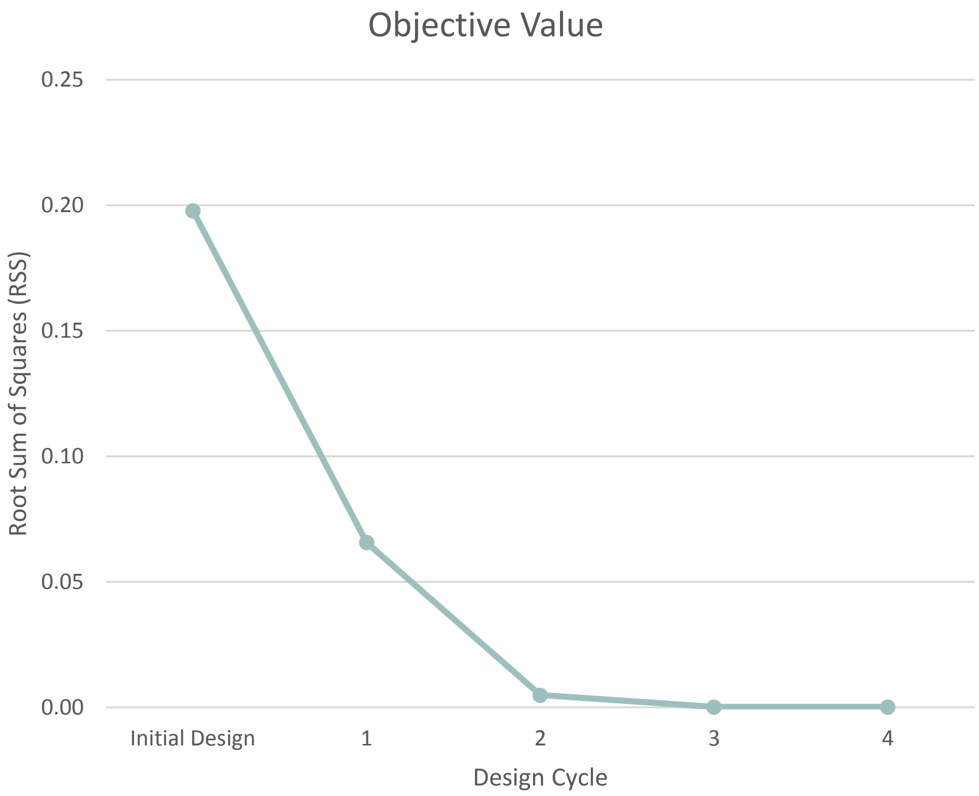
Optimization Results

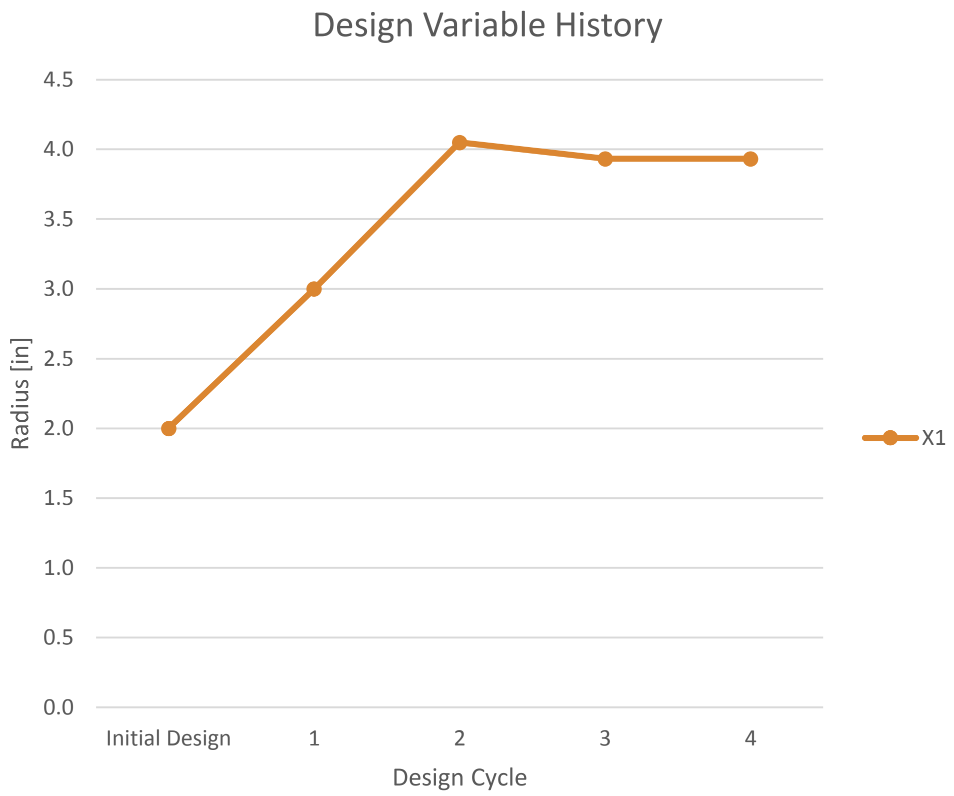
- The following plots show the values for the mode shapes of the initial design and optimized design. Note that the optimized or final design has normal mode shapes that align with experiment





Example 2 - Model Matching / System Identification / Correlation to Experiment

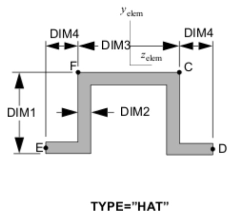




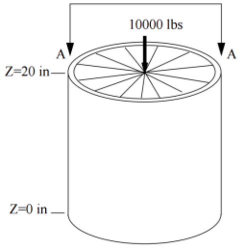
Example 3 – Buckling Optimization of a Thin Walled Cylinder

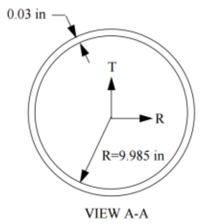
A thin walled cylinder reinforced with ring stiffeners is subjected to an axial compressive load. The initial design is far from exceeding the allowable stresses in the stiffeners and wall. The buckling factor is well above 1, so buckling will not occur.

Optimization is used to vary two structural dimension, wall and stiffener thickness, so as to minimize the mass of the structure while ensuring stress is not exceeded and buckling does not occur. This optimization is an example of multi-discipline optimization since the structure is optimized for both static and buckling analyses.



DIM1: 1 in.
DIM2: .03 in
DIM3: 1 in.
DIM4: .5 in





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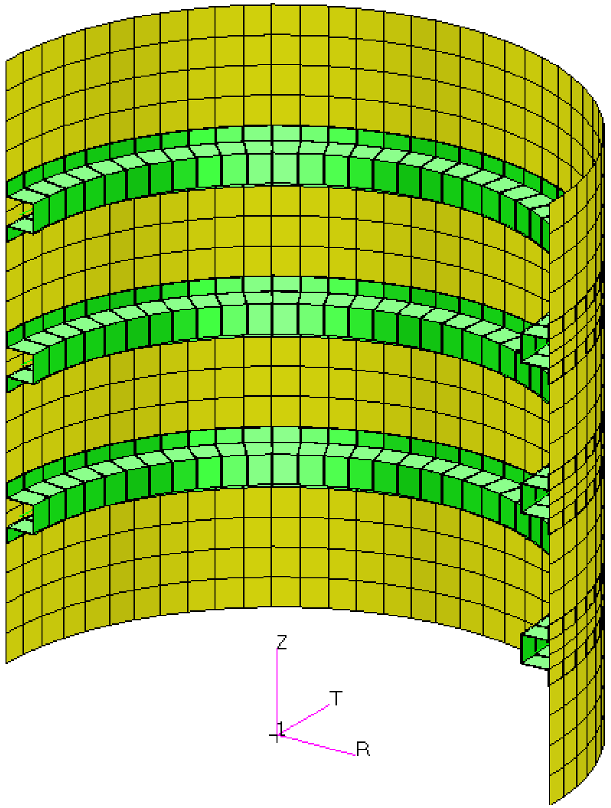
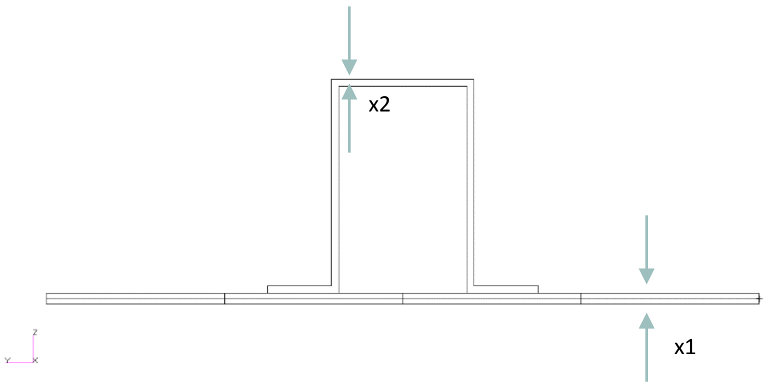


Example 3 – Buckling Optimization of a Thin Walled Cylinder

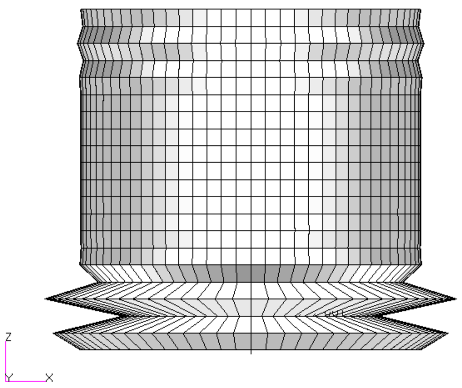
Optimization Problem Statement

- Design Variables:
  - x1: The thickness of the thin wall is allowed to vary. Bounds: .001 < x1 < 10.
  - x2: The thickness of the stiffener cross section is allowed to vary. Bounds: .001 < x2 < 10.
- Design Objective
  - Minimize Weight
- Design Constraints
  - Statics Subcase
    - The maximum beam stress in the stiffener is allowed to be no greater than 25000.
    - The maximum von Mises stress for the z1 and z2 fibers of the thin wall shall be no greater than 25000.
  - Buckling Subcase
    - The buckling load factor shall be no less than 1.0

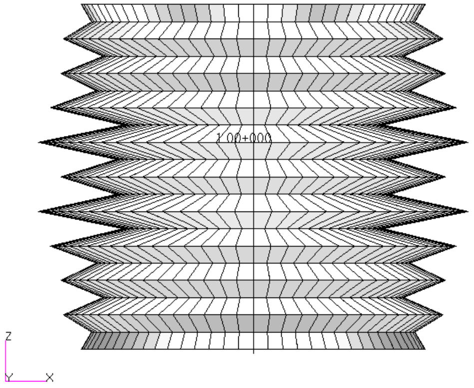


Optimization Results

- Initial Design
  - Mass: 5.76 lbf
  - x1 = Thickness of wall = .03 in
  - x2 = Thickness of stiffener = .03 in
  - Buckling factor = 3.6
  - Stiffener and shell stresses within limits
- Optimized Design
  - Mass: 1.77 lbf
  - x1 = Thickness of wall = .0135 in
  - x2 = Thickness of stiffener = .0016 in
  - Buckling factor = .99 ≈ 1.0
  - Stiffener and shell stresses within limits

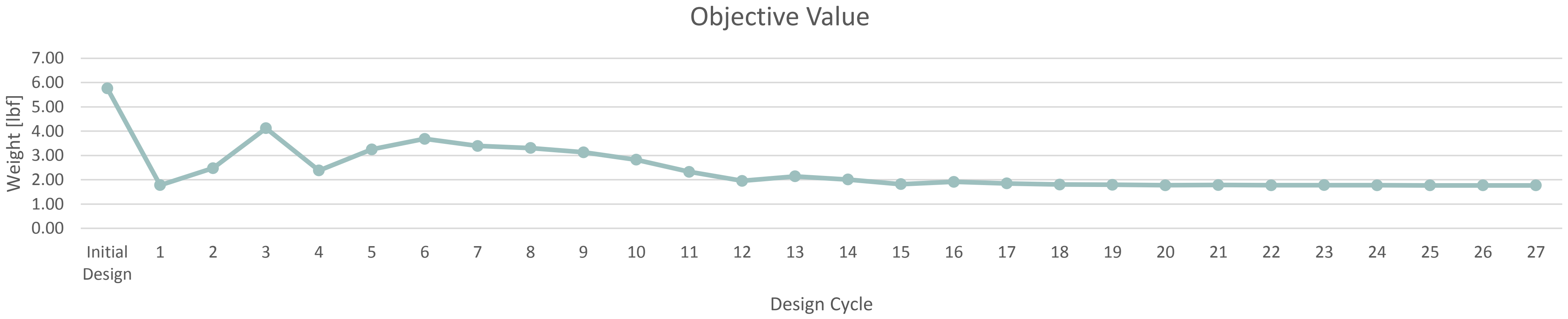


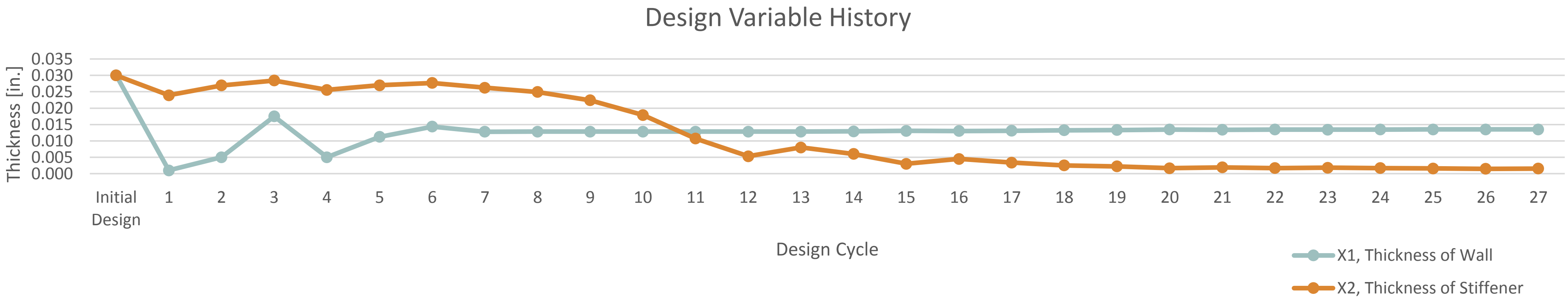
Buckling Shape (B.F. = 3.6)



Buckling Shape (B.F. = .99)

Example 3 – Buckling Optimization of a Thin Walled Cylinder



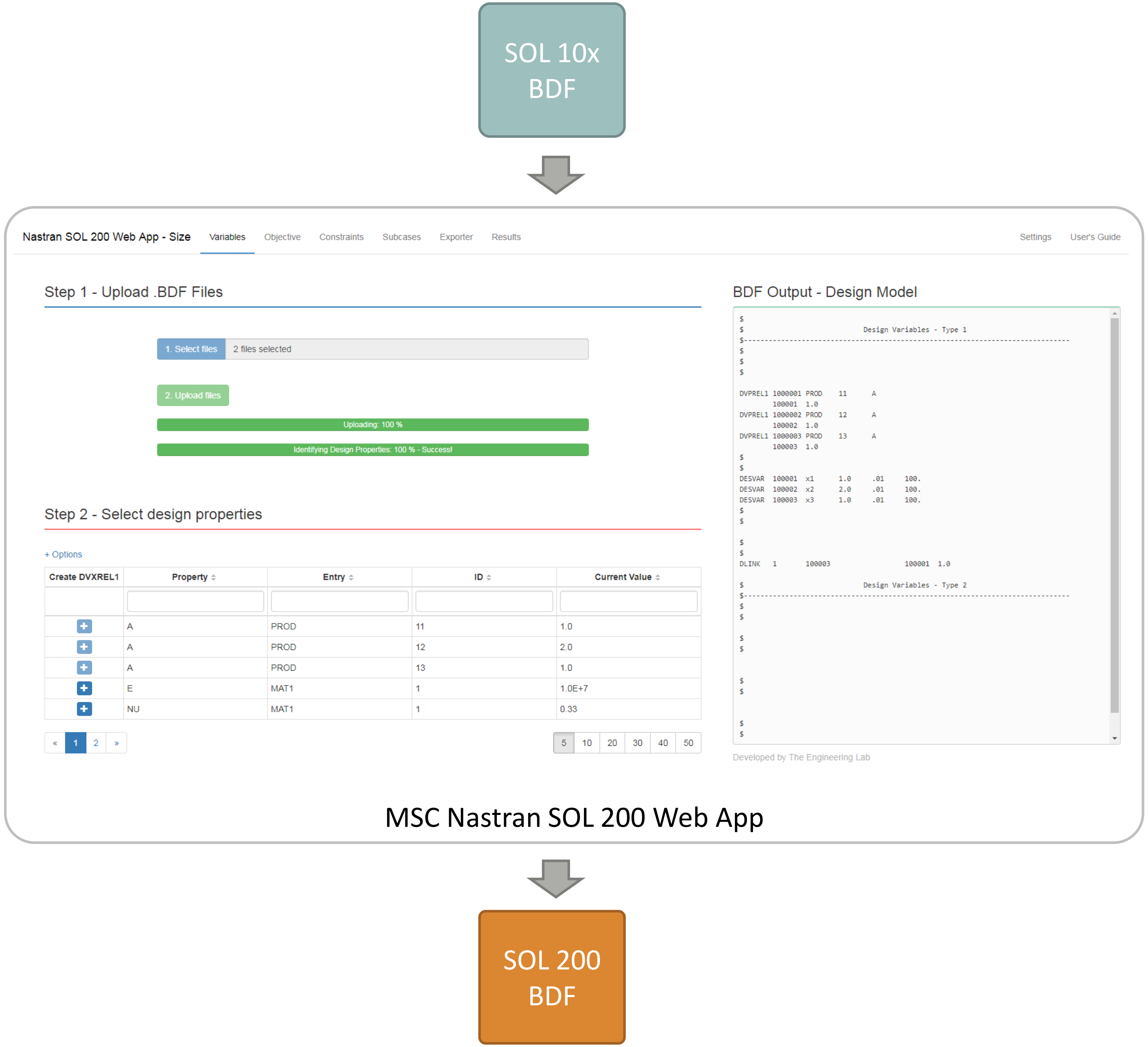


How to Set Up Nastran SOL 200

A STEP-BY-STEP PROCEDURE FOR CONVERTING .BDF OR .DAT FILES TO SOL 200

To perform optimization, your original .bdf or .dat file must be converted to SOL 200

The Nastran SOL 200 Web App facilitates this process



Step 0 – Draft the optimization problem statement

- Design Variables – There are 3 design variables
  - x1: Area of element 1
  - x2: Area of element 2
  - x3: Area of element 3
  - The variables are allowed to vary between .001 and 100.
  - The area of element 3 must equal to the area of element 1
    - x3 = x1
- Design Objective
  - Minimize weight
- Design Constraints
  - g1 - The x and y displacement at node 4 is allowed to be within -.2 and .2 inches
  - g2 - The axial stress in each element is allowed to be within -15000 psi and 20000 psi
  - Both of these constraints are applied to Subcase 1 and 2

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Three-Bar Truss Optimization Example

This example can be found in the MSC Nastran Design Sensitivity and Optimization User's Guide. The text and images below have been extracted from that user's guide.

"A common task in design optimization is to reduce the mass of a structure subjected to several load conditions. Figure 8-1 shows a simple three-bar truss that must be built to withstand two separate loading conditions. Note that these two loads subject the outer truss members to both compressive as well as tensile loads. Due to the loading symmetry, we expect the design to be symmetric as well. As an exercise, we'll show how to enforce this symmetry using design variable linking." - *MSC Nastran Design Sensitivity and Optimization User's Guide, Chapter 8: Example Problems, Three-Bar Truss*

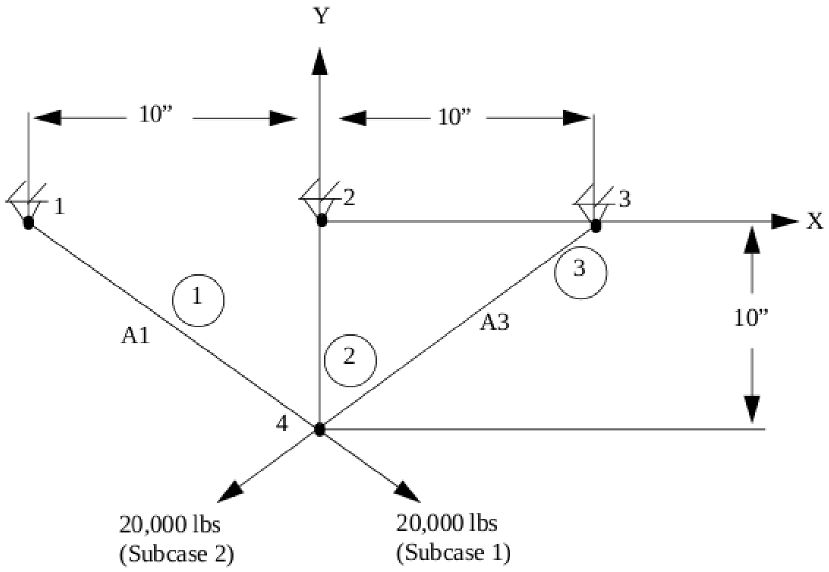


Figure 8-1 Three-Bar Truss

Analysis Model Description

Three rod (pin-jointed) structure in the x-y plane

Symmetric structural configuration with respect to the y axis

Two distinct 20,000 lb load conditions

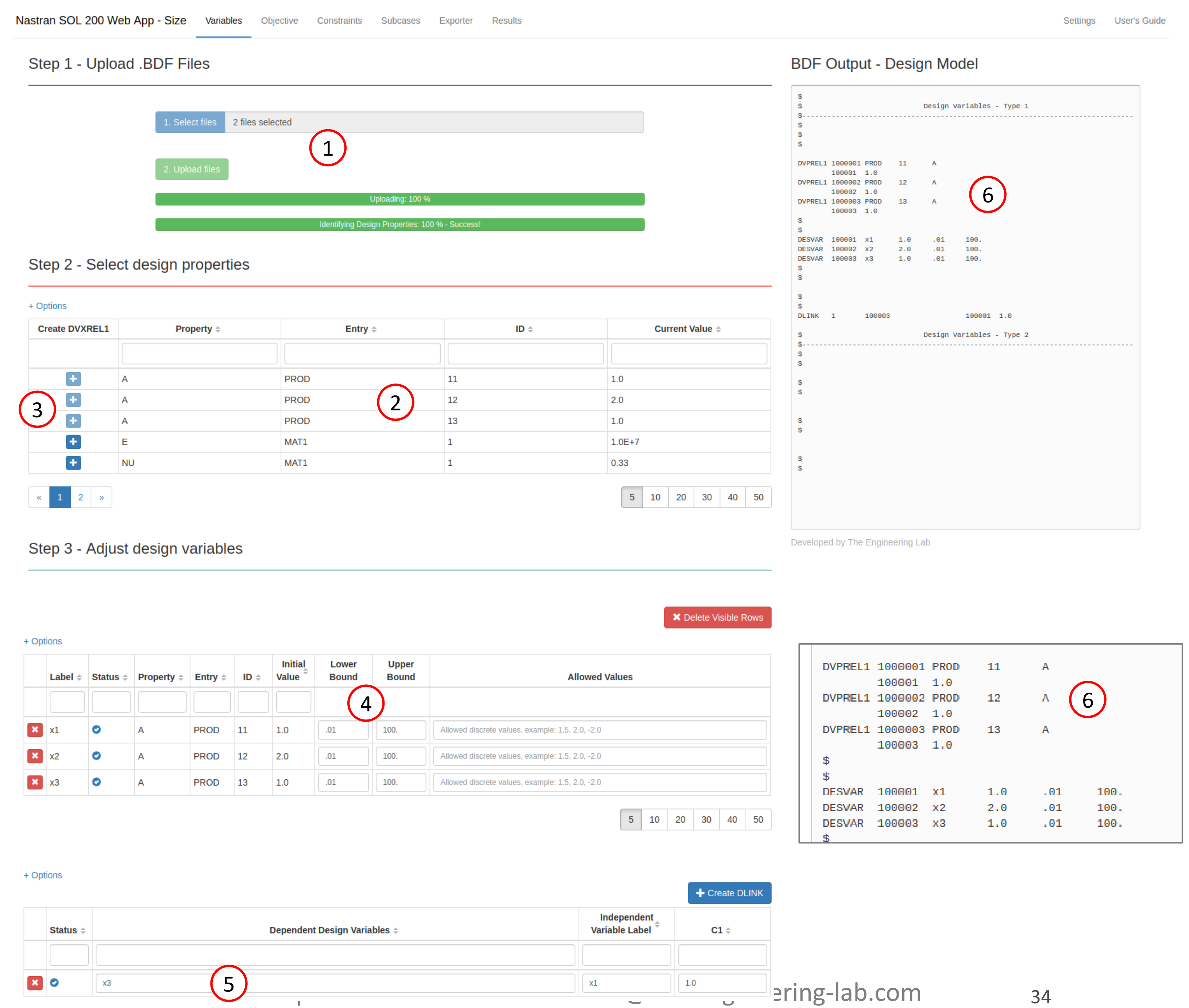
Material:	E = 1.0E7 psi
	Weight density = 0.1 lbs/in^3

Design Model Description

Objective:	Minimization of structural weight	
Design variable:	Cross-sectional areas A_1, A_2, and A_3	
	Design variable linking such that $A_3 = A_1$	
Constraints:	**Allowable stress:**	Tensile 20,000 psi
		Compressive = -15,000 psi
	Displacement:	$\pm$ 0.2 at grid 4 in x end of y directions

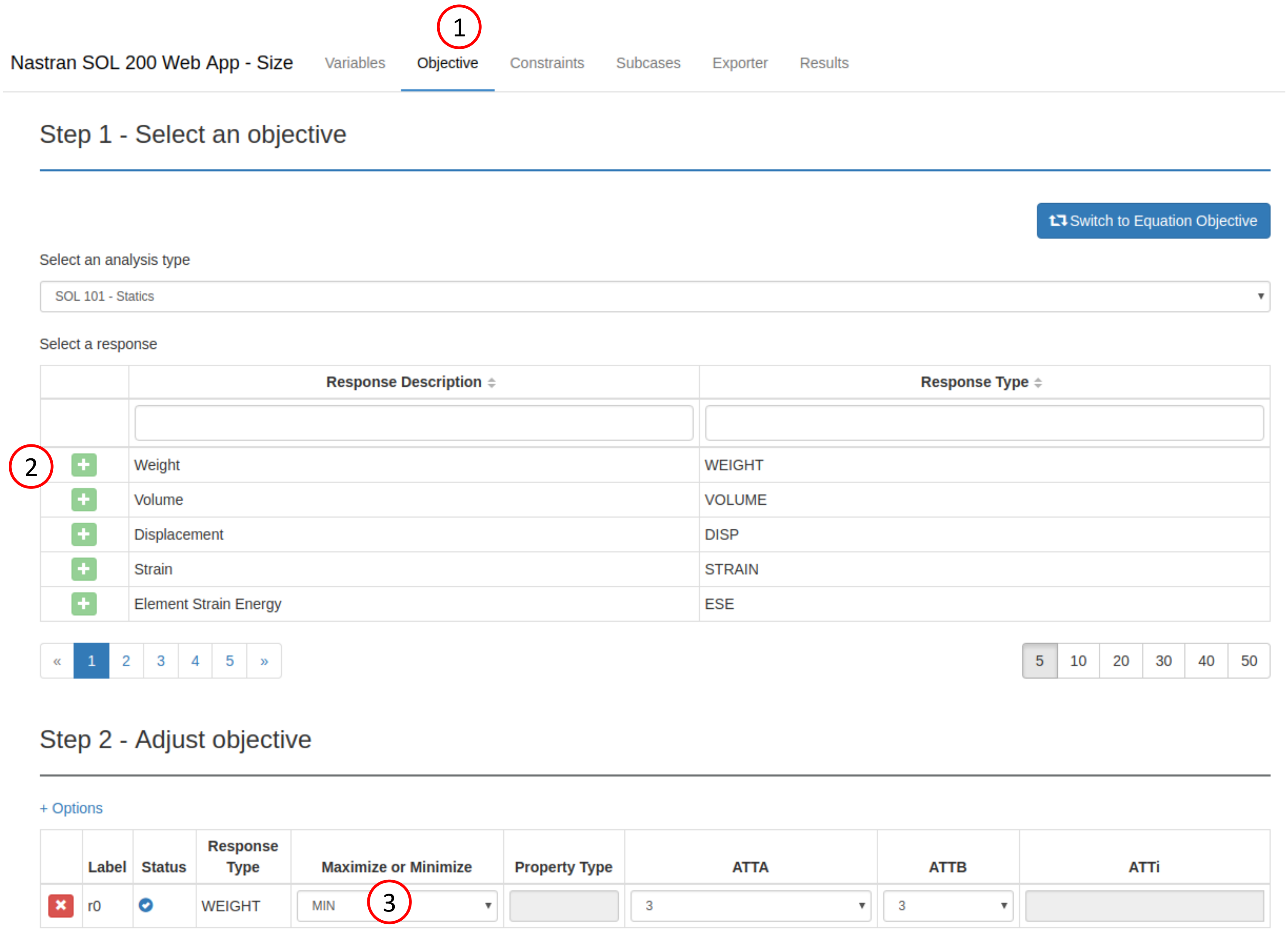
Step 1 – Create Design Variables

1. Select and upload your starting BDF/DAT file
2. Structural parameters, thickness, density, etc., are identified
3. Click the plus (+) icons of the parameters to set the parameters as design variables
4. Adjust the lower and upper bounds of the design variables
5. Create the link between design variables x3 and x1
6. Note that as you use the Web App, the necessary SOL 200 bulk data entries are automatically created



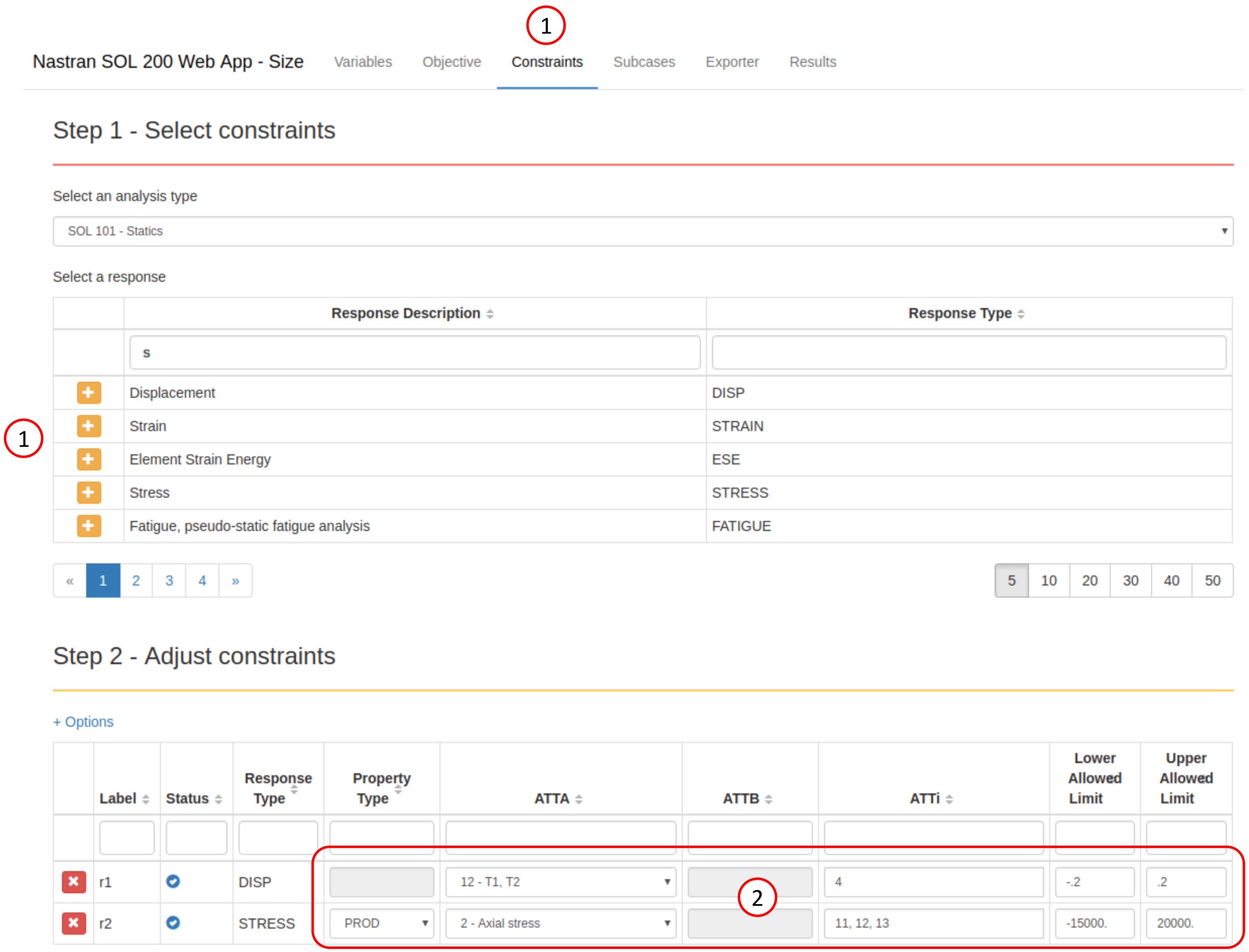
Step 2 – Create Design Objective

1. Navigate to the Objective section
2. Select Weight as the objective from the list of available responses
3. Set the objective to minimize



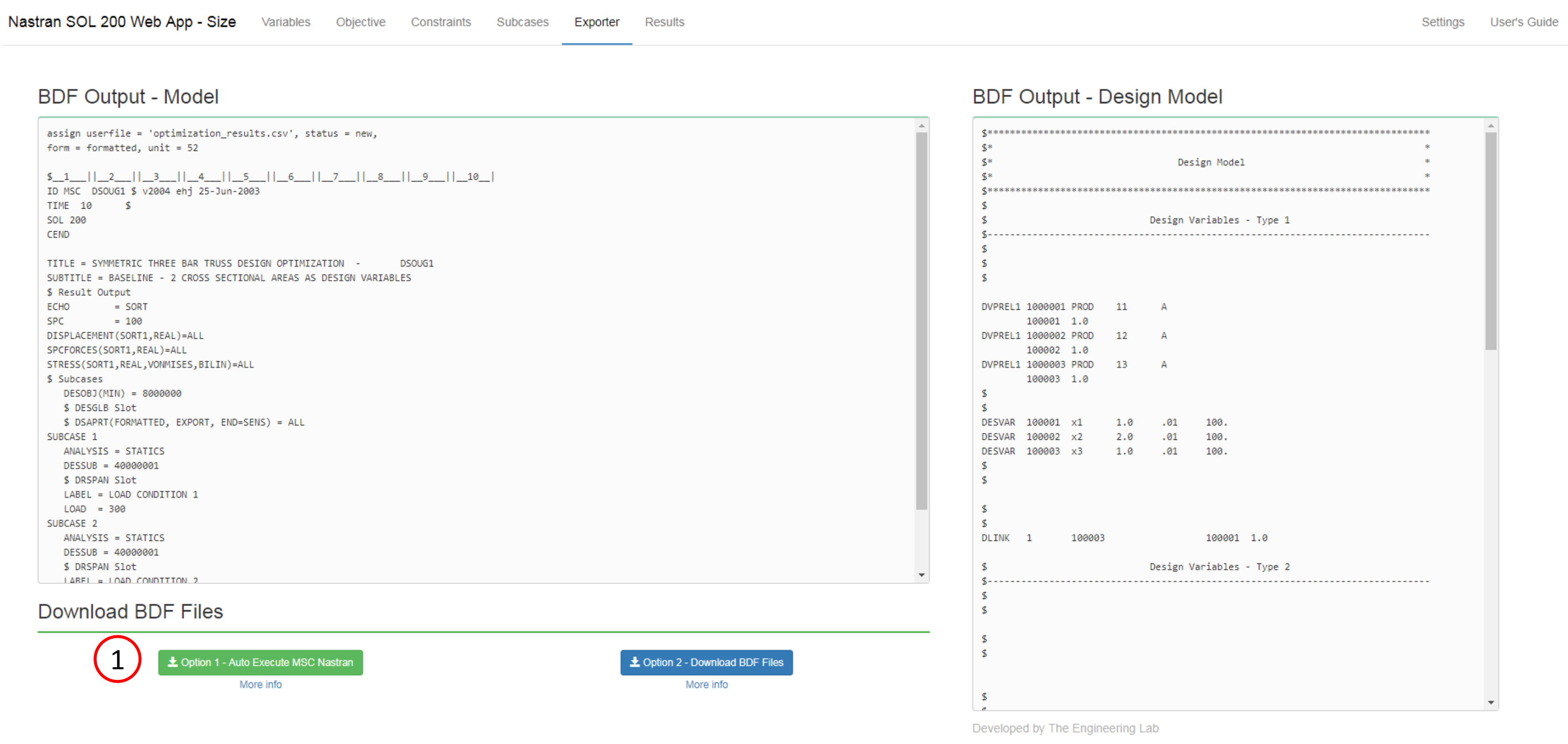
Step 3 – Create Design Constraints

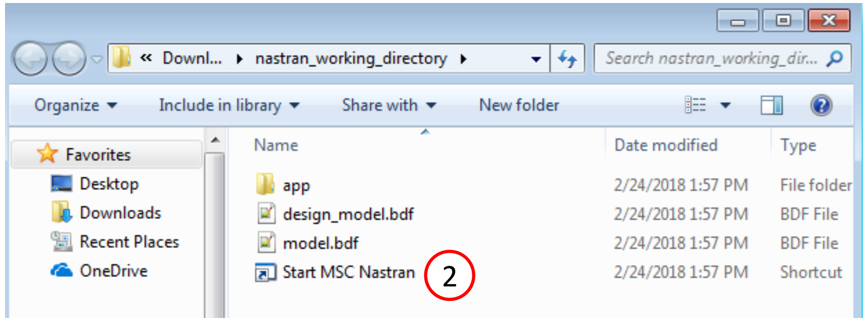
1. Navigate to the Constraints section
2. Click on the plus icons next to Displacement and Stress to create two constraints
3. Configure the Displacement and Stress constraints

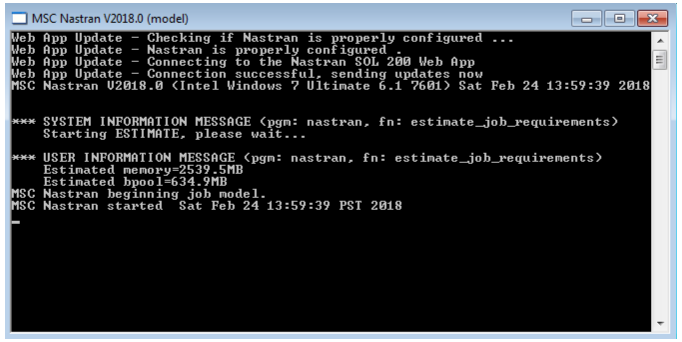


Step 4 – Export

1. Click on Download BDF to download new .bdf files to your desktop
2. Start MSC Nastran to perform the optimization

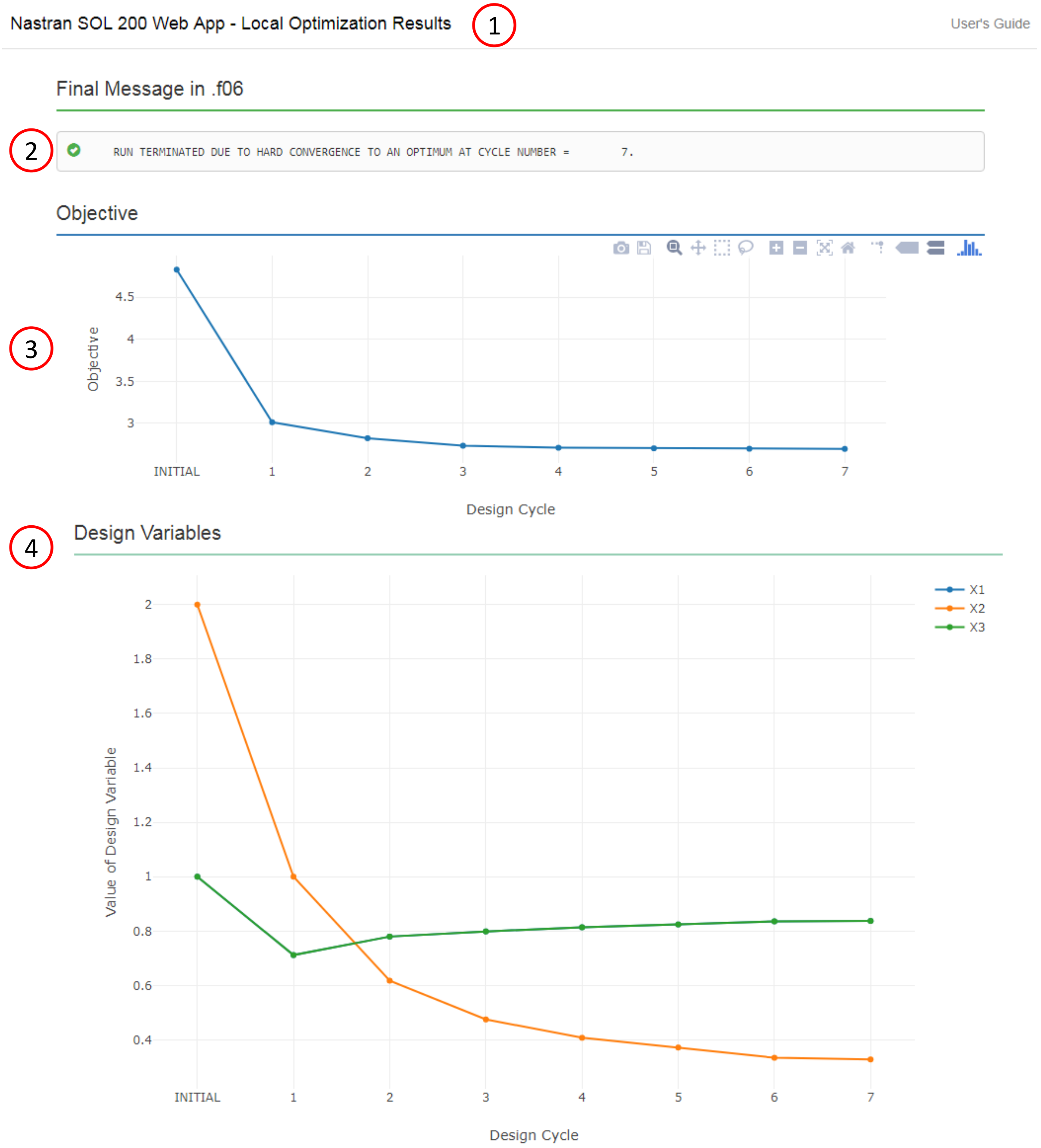






Step 5 - Review the results

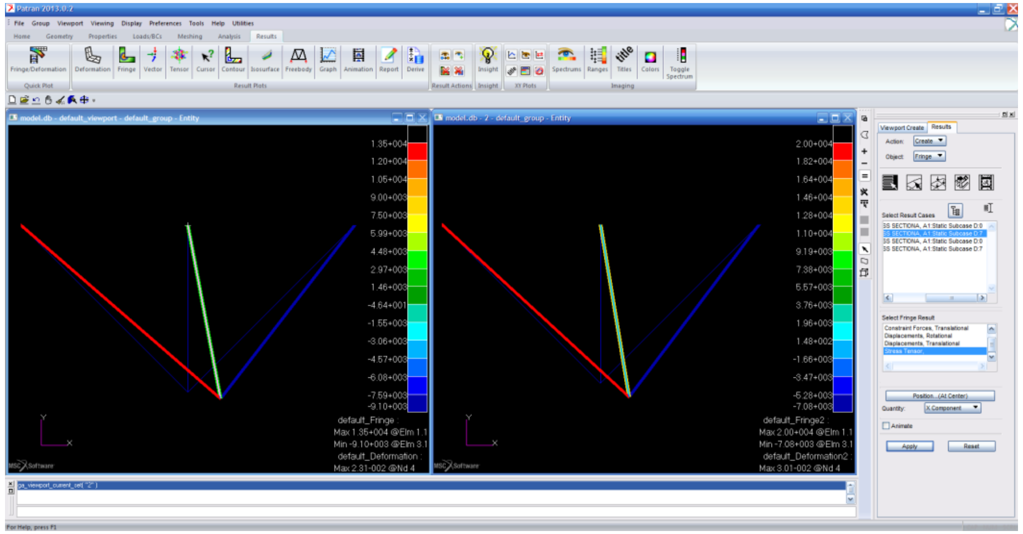
1. The results are automatically displayed once the optimization is complete
2. The status indicates a successful optimization
3. The change in objective can be viewed
4. The change in design variables can be viewed



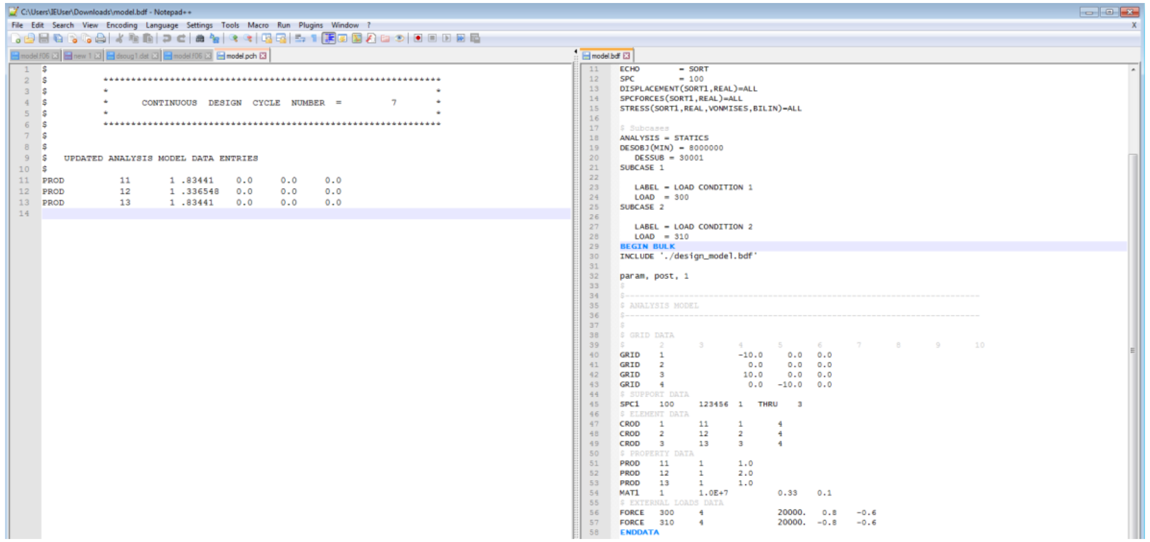
Next Steps

A. The structural results of the optimized design can be viewed in a pre/post processor

B. The original .bdf file can be updated with new optimized entries found in the .pch file



A Patran is used to view structural results. Left: Original structural results of the initial design. Right: New structural results of the optimized design.



B Left: The .pch file has the updated PROD entries. Right: The original .bdf file is updated with optimized PROD entries.

Nastran SOL 200 Learning Resources

Here is a quick summary of helpful tutorials and guides for Nastran SOL 200.

As always, if you have a Nastran SOL 200 question, you are welcome to email me.

Resource	Link
Nastran SOL 200 Tutorials on YouTube Hours of Nastran SOL 200 tutorials are available on my YouTube channel.	Link
Nastran SOL 200 Web App This web application will enable you to convert you existing .bdf files and convert them to SOL 200.	Email me for access
Free Live Training Attend a live training course instructed by me and over 7 hours in length.	Link
Guidance from an Optimization Expert If you are working an optimization project of your own and would like support or guidance, you are welcome to email me.	Email me
MSC Nastran Design Sensitivity and Optimization User's Guide This guide is where I gained a majority of my optimization knowledge and I highly recommend it as a reference.	Link

Final Comments

You are now one step closer to optimizing structures automatically.

I am here to support you, and if you have any questions regarding Nastran SOL 200, you are welcome to email me.

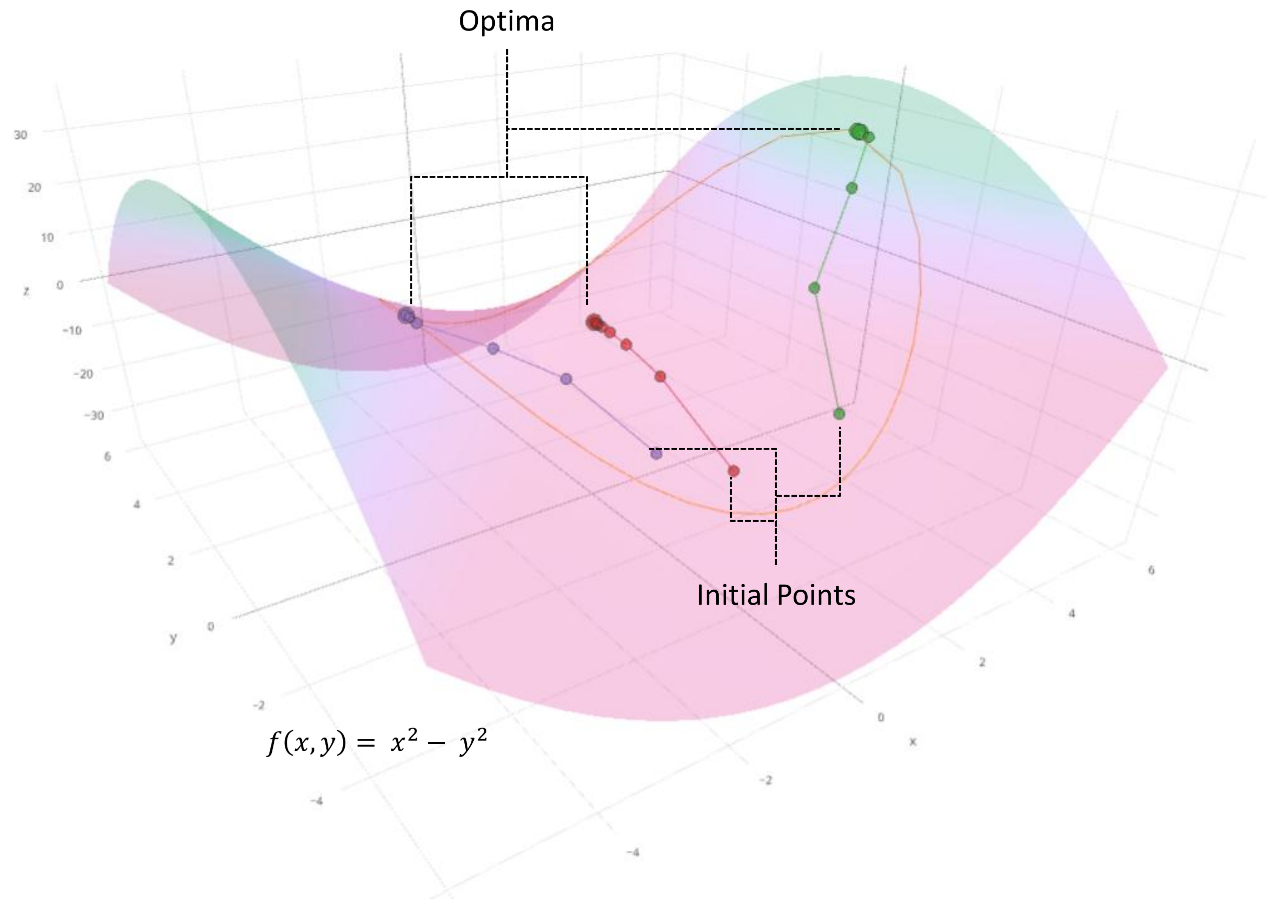
Thank you for reading this guide and stay motivated.

Sincerely,

Christian Aparicio

One last optimization example

The objective is to find the maximum of $f(x,y) = x^2 - y^2$ contained within the ellipsoid. x and y are allowed to vary. Three optimizations were performed, each represented by the colors purple, red and green. Each optimization had a different initial point. In each scenario, the optimizer followed the path of steepest accent (highest gradient), but led to 3 different maxima. When finding optimums of functions with higher dimensions, keep in mind that multiple optimums may exist and that each optimum will depend on your initial design variables.



Bonus Section – What is Sensitivity Analysis?

Sensitvity analysis the process of calculating partial derivatives.

The partial derivatives are commonly referred to as sensitivity coefficients or just sensitivities when using Nastran SOL 200.

Since sensitivities are nothing more than rates of change, design variables can be selected based on their influence on outputs. A question such as this can be answered, which design variable, x1 or x2, if varied, will best minimize the function f? Another example, if the goal is to minimize a weight function, it would be logical to select design variables that have sensitivities to the weight.

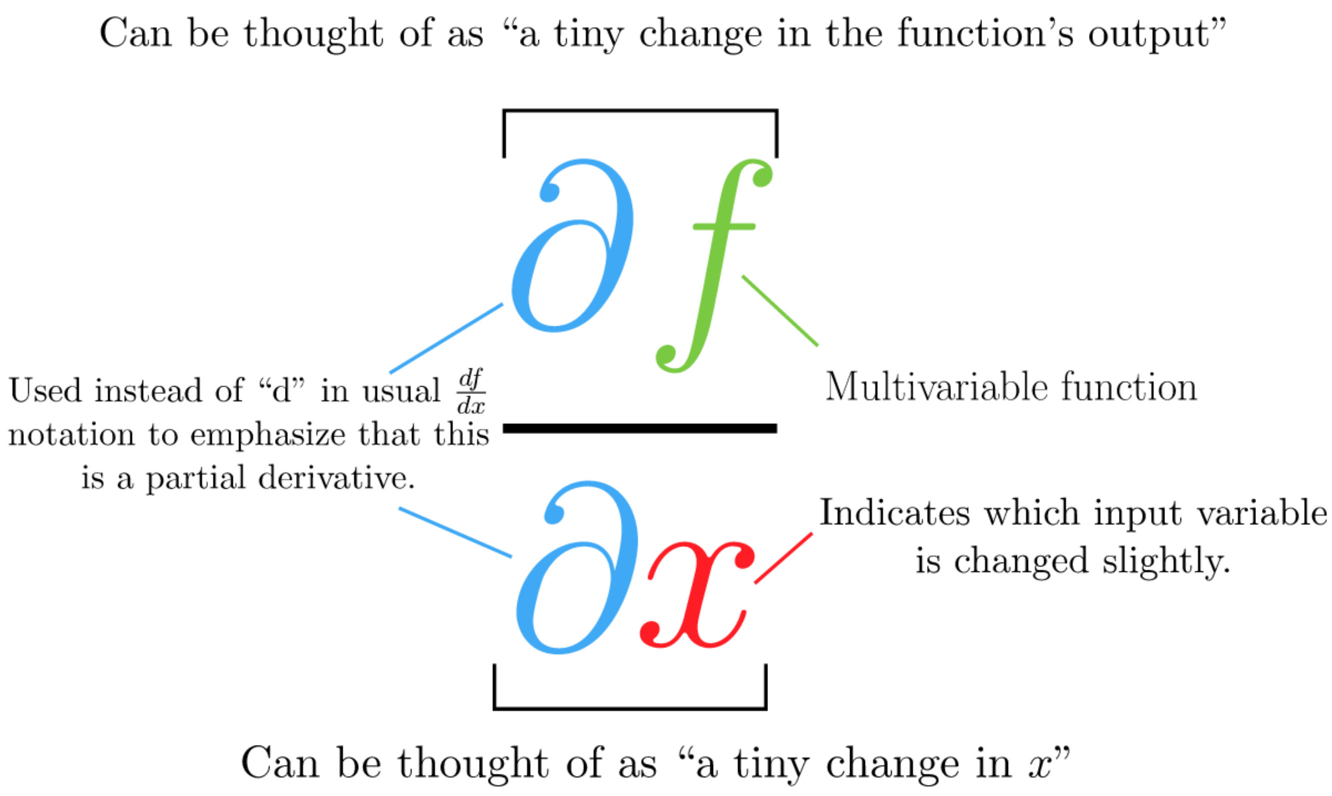
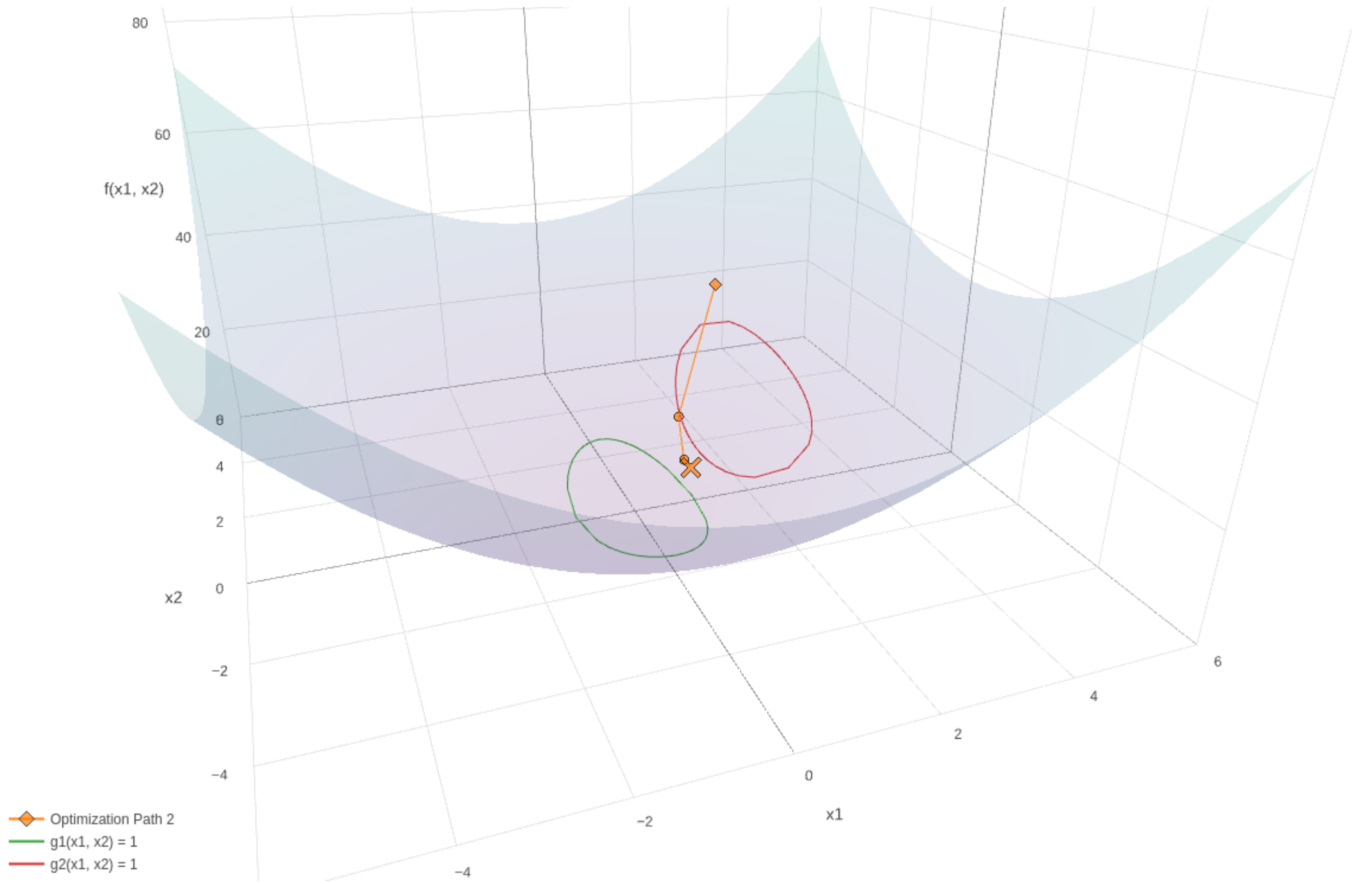


Image source: [khanacademy.org – Introduction to Partial Derivatives]

Let us consider a simple example as shown to the right.

Relevant Equations	Optimization Problem Statement
$f(x_1, x_2) = x_1^2 + x_2^2$ $g_1(x_1, x_2) = x_1^2 + (\frac{x_2}{2})^2$ $g_2(x_1, x_2) = (x_1 - 2.5)^2 + (\frac{x_2}{2} - .75)^2$	Objective: Minimize $f(x_1, x_2)$ Initial Point: $(x_1, x_2) = (3, 4)$ Constraints: $g_1(x_1, x_2) \leq 1$ $g_2(x_1, x_2) \leq 1$



The sensitivities are computed by hand and will be compared to the sensitivities computed by Nastran SOL 200.

Relevant Equations	Optimization Problem Statement
$f(x_1, x_2) = x_1^2 + x_2^2$ $g_1(x_1, x_2) = x_1^2 + (\frac{x_2}{2})^2$ $g_2(x_1, x_2) = (x_1 - 2.5)^2 + (\frac{x_2}{2} - .75)^2$	Objective: Minimize $f(x_1, x_2)$ Initial Point: $(x_1, x_2) = (3, 4)$ Constraints: $g_1(x_1, x_2) \leq 1$ $g_2(x_1, x_2) \leq 1$

$$\frac{\partial f}{\partial x_1} = 2x_1 \qquad \frac{\partial g_1}{\partial x_1} = 2x_1 \qquad \frac{\partial g_2}{\partial x_1} = 2(x_1 - 2.5)$$

$$\frac{\partial f}{\partial x_1}\Big|_{(3,4)} = 6 \qquad \frac{\partial g_1}{\partial x_1}\Big|_{(3,4)} = 6 \qquad \frac{\partial g_2}{\partial x_1}\Big|_{(3,4)} = 1$$

$$\frac{\partial f}{\partial x_2} = 2x_2 \qquad \frac{\partial g_1}{\partial x_2} = \frac{x_2}{2} \qquad \frac{\partial g_2}{\partial x_2} = \frac{x_2}{2} - .75$$

$$\frac{\partial f}{\partial x_2}\Big|_{(3,4)} = 8 \qquad \frac{\partial g_1}{\partial x_2}\Big|_{(3,4)} = 2 \qquad \frac{\partial g_2}{\partial x_2}\Big|_{(3,4)} = 1.25$$

This is the sensitivity output by Nastran SOL 200 and can be found in the .f06 file.

Note that Nastran SOL 200 produces sensitivities in line with the hand calculated sensitivities.

```
 ****************************************************************************
 *                                                                          *
 *   D E S I G N   S E N S I T I V I T Y   M A T R I X   O U T P U T        *
 *                                                                          *
 *                                                                          *
 *   R E S P O N S E   S E N S I T I V I T Y   C O E F F I C I E N T S      *
 *                                                                          *
 ****************************************************************************
1                                         JANUARY  12, 2017  MSC Nastran 11/13/14   PAGE    16
0
 ---------------------------------------------------------------------------------------------------------------
 DRESP2 ID=    9000000          RESPONSE TYPE= SYNTHETIC                                       SEID=         0
 SUBCASE  RESP VALUE  FREQ/TIME   DESIGN  VARIABLE  COEFFICIENT   DESIGN  VARIABLE  COEFFICIENT   DESIGN  VARIABLE  COEFFICIENT
 ---------------------------------------------------------------------------------------------------------------
       0  2.5000E+01  0.0000E+00        4 DUM        0.0000E+00   100001  X1          5.9995E+00   100002  X2          8.0000E+00
```

$f(3,4) = 25$ $\qquad \frac{\partial f}{\partial x_1}\Big|_{(3,4)} = 6$ $\qquad \frac{\partial f}{\partial x_2}\Big|_{(3,4)} = 8$

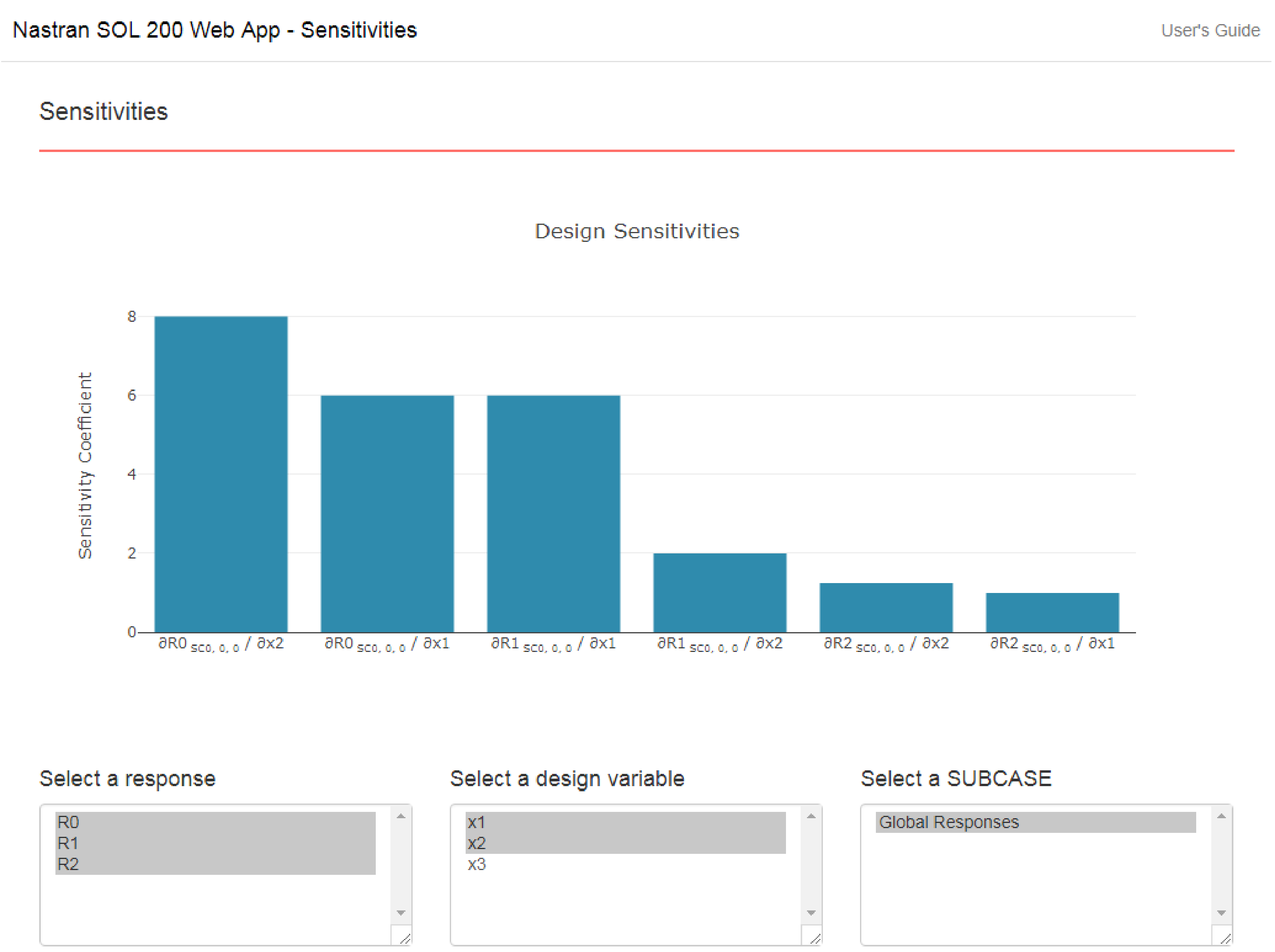
```
 ---------------------------------------------------------------------------------------------------------------
 DRESP2 ID=    9000001          RESPONSE TYPE= SYNTHETIC                                       SEID=         0
 SUBCASE  RESP VALUE  FREQ/TIME   DESIGN  VARIABLE  COEFFICIENT   DESIGN  VARIABLE  COEFFICIENT   DESIGN  VARIABLE  COEFFICIENT
 ---------------------------------------------------------------------------------------------------------------
       0  1.3000E+01  0.0000E+00        4 DUM        0.0000E+00   100001  X1          6.0000E+00   100002  X2          2.0000E+00
```

$g_1(3,4) = 13$ $\qquad \frac{\partial g_1}{\partial x_1}\Big|_{(3,4)} = 6$ $\qquad \frac{\partial g_1}{\partial x_2}\Big|_{(3,4)} = 2$

```
 ---------------------------------------------------------------------------------------------------------------
 DRESP2 ID=    9000002          RESPONSE TYPE= SYNTHETIC                                       SEID=         0
 SUBCASE  RESP VALUE  FREQ/TIME   DESIGN  VARIABLE  COEFFICIENT   DESIGN  VARIABLE  COEFFICIENT   DESIGN  VARIABLE  COEFFICIENT
 ---------------------------------------------------------------------------------------------------------------
       0  1.8125E+00  0.0000E+00        4 DUM        0.0000E+00   100001  X1          1.0000E+00   100002  X2          1.2500E+00
```

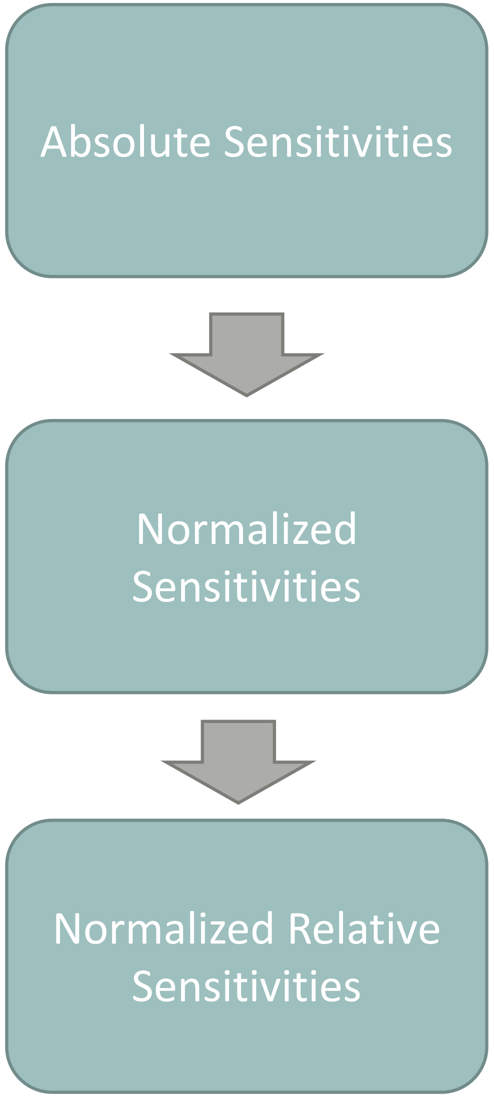
$g_2(3,4) = 1.8125$ $\qquad \frac{\partial g_2}{\partial x_1}\Big|_{(3,4)} = 1$ $\qquad \frac{\partial g_2}{\partial x_2}\Big|_{(3,4)} = 1.25$

The same sensitivities can be automatically plotted in the Nastran SOL 200 Web App



There are different types of sensitivities you may have to consider.

To the right are some sensitivities and how they are related.



$$\frac{df}{dx_1} = 6$$

$$\frac{df}{dx_1}x_1 = 6 \times 3$$

$$= 18$$

$$\frac{df}{dx_1}\frac{x_1}{f(x_1,x_2)} = 6 \times \frac{3}{25}$$

$$= .72$$